USMA Problem of the Week #15
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Consider the eight instructors as distinguishable balls, and their scores as distinguishable boxes. There could be anywhere from one box (all eight get the same score) to eight boxes (all eight get different scores). The number of ways that n distinguishable balls can be put into m distinguishable boxes, with at least one ball in each box, is $m! \cdot \left\{ {n \atop m} \right\}$, where $\left\{ {n \atop m} \right\}$ is the Stirling number of the second kind. Thus the answer to our problem is the sum $\sum_{m=1}^{8} m! \cdot \left\{ {8 \atop m} \right\} = 545{,}835$.

Note: Stanley's Twelvefold Way is a convenient method of considering the basic ways in which balls can be put into boxes. The balls can be distinguishable or indistinguishable, as can the boxes. The (implied) function from balls to boxes can be injective, surjective, or have no restriction. This gives 12 different conditions for the ball/box problem. The formula above is for distinguishable balls (we can tell the instructors apart) and distinguishable boxes (we can tell a higher score from a lower score), and a surjective function. Running the summation from 1 through 8 gives all the possible number of distinct scores for the instructors. A handy summary of the Twelvefold Way can be found at http://mathsci.kaist.ac.kr/~drake/pdf/twelvefold-way.pdf.