Problem of the Week #16
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We can model the spiral groove by the polar function $r = \frac{\theta}{156\pi}$ inches, as θ runs from $2\pi \cdot 156$ to $2\pi \cdot 390$. This function gives an initial radius of $r = 2$ inches and a final radius of $r = 5$ inches, and $3 \cdot 78$ revolutions. (Each time θ increases by 2π, the spiral makes another revolution, and the value of r increases by $1/78$ inches.) The arc length of a polar curve is $L = \int_{\theta=\alpha}^{\theta=\beta} \theta\sqrt{r^2 + (r')^2}\, d\theta$, which in this case is $\frac{1}{156\pi}\int_{312\pi}^{780\pi} \sqrt{\theta^2 + 1}\, d\theta = 5145.93$ inches.

The rotational speed of an LP is constant, which means that halfway through the song the needle is halfway between 2 inches and 5 inches, or 3.5 inches from the center. (The linear speed of the needle in the groove constantly decreases as the LP is played, but it takes the same time for the LP to turn once wherever the needle is. Modern CDs maintain a constant linear speed, which means a changing rotational speed.) That of course is much different than the point at which the needle is halfway through the entire length of the groove. This event happens when $\frac{1}{156\pi}\int_{312\pi}^{156k\pi} \sqrt{\theta^2 + 1}\, d\theta = \frac{5145.93}{2}$, with k being the number of inches from the center to the needle. My TI84+ gives a value of $k = 3.81$ inches for this quantity.