

Problem of the Week #6

Assigned: 7 OCT 10

Due: 14 OCT 10

Alphabetizing

MAJ Arney records test grades by first putting all tests in alphabetical order and then inputting the grades into the online system. In order to alphabetize the tests, MAJ Arney starts by comparing the first two tests. If they are not in alphabetical order, she interchanges their positions, otherwise, she looks at the next pair of tests (i.e., tests 2 and 3), continuing in this manner until she finds a pair that is out of order. Each time she comes to a pair of tests that is not alphabetized, she interchanges them and then returns to the beginning of the stack and starts the process over. When she gets through all the tests without finding any pairs out of order, she knows the tests are alphabetized.

MAJ Arney has a total of n students, and she receives all of the tests in a random order. What is the maximum number of interchanges MAJ Arney will perform before all of the tests are in order?

Solution: The maximum number relates to the case when the tests are in reverse alphabetical order, and every pair of tests must be interchanged. The number of pairs of test that exist out of n tests is $\binom{n}{2}$.

Email solutions to Christopher.marks@usma.edu with subject line: POTW.