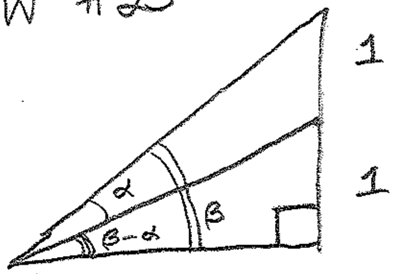
POTW #23



$$\tan\beta = 2/x \qquad (1)$$

$$\tan(\beta-\alpha) = 1/x \qquad (2)$$

$$\tan(\beta-\alpha) = \frac{\tan\beta - \tan\alpha}{1+\tan\beta\tan\alpha} \qquad (3)$$

Combining (1) and (2) w/ trig. identity (3), we get

$$\frac{1}{x} = \frac{2/x - \tan\alpha}{1 + 2/x\,\tan\alpha} = \frac{2 - x\tan\alpha}{x + 2\tan\alpha}$$

Now solve for $\tan\alpha$:

$$x + 2\tan\alpha = 2x - x^2\tan\alpha$$

$$\Rightarrow \quad \tan\alpha = \frac{x}{2+x^2}$$

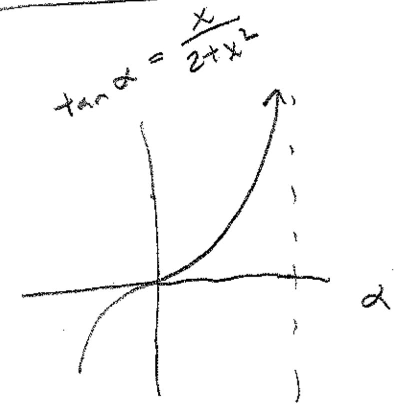


Fig 1 $\pi/2$

From Fig 1 we see α is maximized exactly when $\tan\alpha$ is (b/c $\tan\alpha$ is increasing on $[0, \pi/2)$)

So let $f(x) = \dfrac{x}{2+x^2}$.

$$f'(x) = \frac{2-x^2}{(2+x^2)^2}$$ so f has a max at $\underline{\underline{x = \sqrt{2}}}$.

(The max value of $\tan\alpha$ is $\dfrac{\sqrt{2}}{4}$, so the max angle is $\alpha = \arctan\left(\dfrac{\sqrt{2}}{4}\right) \approx 19.4$ degrees)

$$\boxed{x = \sqrt{2} \text{ units}}$$

– Wooster