Mathematica Militaris

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OF THE FEDERAL SERVICE ACADEMIES

''Old school vs. new school'' Does it make a difference?

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EDITOR'S NOTES

I thoroughly enjoyed reading the submissions for this edition of Mathematica Militaris. Our faculty has some strong opinions on the topic of "Old vs. New", and they presented for you here, as well as some tremendous ideas on how to leverage this technology to provide the best possible education for our math students.

The lead article is from the former Head of the USMA Department of Mathematical Sciences, Brigadier General (Retired) Chris Arney. BG Arney eloquently points out that through the use of technology we have more options and reach our students. His contribution provides guidelines that remain valuable even as we and our students have increased access to information technology. I'm certain you will gain valuable insight from his observations and opinions.

Lieutenant Colonel Scott Billie provides a unique article that parallels the experiences and environment of the beginning student in mathematics to those of the officer confronted with the high-tech battlefield in today's military units, the "Objective Force". It most certainly has an Army flair to it, but I'm confident the civilians and officers from the sister services can easily relate to his examples.

The next article, by Major Paul Goethals also has an Army flavor to it, but MAJ Goethals uses examples from our leadership and tactical reference manuals. He presents a nice argument that emphases the importance of technology in our classrooms and the need to remain on the "cutting edge". However, MAJ Goethals does not completely disregard traditional methods and does well to point out some caveats and precautions as we continue move forward with new technologies.

In their contribution, Dr. Jim Rolf and Dr. Michael Brilleslyper of the USAFA Department of Mathematical Sciences provides some cogent observations from their experiences incorporating technology into the core calculus sequence at the Air Force Academy. They tackle some tough issues like the new set of skills required of future officers and the proper balance of theory and application.

The final article is written by Professor Brian Winkel of the USMA Department of Mathematical Sciences. Dr. Winkel describes some concrete examples of technology empowering students to investigate and explore mathematics in the classroom and beyond. I have seen Dr. Winkel in the classroom with students and have tried several of these techniques myself. I'm thrilled to see them included in this issue so that others may benefit as well.

In the coming semesters there will be more opportunities for you to read and enjoy Mathematica Militaris. I would encourage you to consider contributing an article to share with your peers. As you read through the contributed papers in this issue, I hope you are inspired to share your own ideas, techniques and strategies with your cohorts.

Be sure to visit our website for past issues: http://www.dean.usma.edu/math/pubs/mathmil/ .

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How We Teach Does Make a Difference

BG(R) Chris Arney, Dean of the School of Mathematics and Science, The College of Saint Rose

A few things come to mind as I think about bridging the generation gap in teaching methodology (old style vs. new styles). The one thing that has changed from my days as a student to now is that we now recognize that students can and do learn through different means. In my days as a student, you either learned by the method that the professor or school used, or you were left behind. We have developed more options, and I think that is good. I don't think anyone neglects the old methodologies, we just add to them. I won't call any of this pedagogy, because as a West Point and RPI graduate, I never was taught what that meant. I won't call these principles either, but they are things about teaching that I think we should continue to think about as teachers.

1) How we teach is related to what we teach. Modeling, Proving, Problem Solving, and Inquiry are mathematical verbs and in courses where these actions are goals, the students must practice these things more often than the teacher demonstrates them. In a body of knowledge or skill course, the teacher may be effective by lecturing and demonstrating more often than having the students practice or drill in class.

2) Balance the use of teaching tools familiar to the students with the diversity of newly available teaching/learning tools. For example, you might use calculators because the students are comfortable with them and a computer software package because it's new, exciting, and effective. Not all your students are going to learn like you did or with the one style or technology that you are best at using.

3) Don't just show the students what you can do or what you know. It won't be enough. Sure being a good role model of a mathematician, thinker, problem solver, modeler, or proof maker is fine and necessary, but the ultimate goal is to make your students better than you. Motivate them to build their skills beyond what you are showing them. I love it when a student solves a problem in a way that I would never think of or brings to class ideas that are beyond the class goals or things that I don't know.

4) Tailor the lesson(s) to the goal(s). Lecturing may be an effective way to build a body of knowledge. Usually discussion is better, and both need to complement reading. Getting your students to read and/or think is very important. What happens outside of class is usually more important than what happens in the classroom. If nothing happens outside of class, no matter how great it is in class, student learning is suffering.

5) Assessment is still linked to how students learn. It doesn't hurt to use assessments that best reflect how you are teaching (e.g., don't give multiple choice tests when you are teaching the mathematical verbs).

6) Finally, try to adapt your teaching styles to fit the needs of your students in each specific course. Sometimes that means changing the plan, being flexible, getting to know the students' needs and learning styles, and, finally, knowing your own capabilities.

I do think that how we teach is as important as what we teach. I personally strive to make every new section that I teach the best that I ever taught. While I have made my own measure of my success, my score is immaterial compared to the students' perspectives. I am also pleased that I have added a few teaching methods to my arsenal since my first days of teaching LTC Horton's USMA classmates back in 1980. Yet if I ever have to resort to "question boards," I could do it as well as I did in the golden age of teaching 4 sections of cadets, 85 minutes per day, 6 days per week. [Whatever happened to Saturday classes?]

Trials and Tribulations of Transforming to the "Objective Force" Classroom

Lieutenant Colonel J. Scott Billie, USMA, Department of Mathematical Sciences

Much like the soldiers and officers participating in the operational force transformation effort, students are undergoing a transformation of their own in classrooms. Just as information flows in from the digital battlefield, an overwhelming amount of technology flows into the classroom. In the recesses of my memory, I can remember my biggest technology challenge was ensuring my calculator had enough charge to last through a class period. Today, twenty years later, our students are equipped with the latest technology and software. I can only imagine these 21st century students going through their own versions of PCIs (Pre-Combat Inspections – or in this case "Pre-Calculus") with the contents of their rucksack (read cadet backpack) laid out on their barracks' floor…

- Dell Latitude Laptop, with extra battery pack – Check
- Microsoft Excel- Check
- Mathematica with functioning password – Check
- Texas Instruments, TI-89 Calculator – Check
- Palm m515, PDA – Check

I can only image the electronic footprint generated by our new electronic classrooms! Among other items I also require students to bring are issued textbooks, notepaper, and writing implements. I make light of the plight of the modern age classroom warriors, but can certainly appreciate the information and technology blitz they are exposed to and expected to learn and master.

I am currently teaching MA101, Introduction to Calculus. Students in this MA100/101 track are selected by a screening process that accounts for high school math backgrounds, standardized college entrance exams and the score on a fundamental skills exam (FSE) administered prior to the start of the academic year by the mathematics department. Based on empirical data and my daily interaction with the cadets in this program, I believe the population of the class appears to be bimodal. One group of students had very little math in high school, but are quick to grasp the concepts, while the other group consists of individuals with an aversion to math in general. Herein lies the problem of bringing technology into the classroom. Which group should your daily lesson planning target? How much time do you devote to fundamental calculus verses the capabilities found on your laptop? We tried to informally answer these questions back in January prior to Lesson 1.

As a low density course, MA101 only has three instructors. As luck would have it, we all have similar thoughts and philosophies on instruction (I assume not a coincidence). Prior to the start of the semester, the course director decided to concentrate our technology efforts to the uploaded software found on the cadets' laptops. Primary emphasis would be placed on using Microsoft® Excel for numerical solutions to Systems of Discrete Dynamical Systems (DDS) and Mathematica® for matrix operations and calculus. Our philosophy would be to answer any questions on use of the TI-89 calculator (most of our students had attended the United States Military Academy Preparation School (USMAPS) at Fort Monmouth and had become very familiar with their operation), but would not dedicate classroom time for calculator instruction.

As the semester progressed, I found myself consistently achieving lesson objectives during classroom instruction, but usually being forced to only demonstrate technology to students. Computer savvy individuals quickly saw the great potential, refined their technology skills during their own time and most often exceeded course standards. Unfortunately, I would say the majority of students struggled to master these skills and only became interested when a graded assignment required the use of technology.

As far as answering the previously mentioned questions, my opinions shifted throughout the semester. Question 1, Which group should your daily lesson planning target? Originally, I thought I could orient instruction towards the center or average needs of the class. After the first few lessons, I realized I was missing both groups and shifted more towards the lowest common denominator. While sending cadets up to the boards for exercises, I provided additional and more complex problems to the students who had a firm grasp of the material. I also encouraged the individuals who "got it" to assist their classmates in understanding certain key concepts. I often found myself using "vertical" explanations to questions. I would end up explaining in greater detail, which sometimes added to increased confusion. Other students however, will often use "lateral" explanations, where they will explain based on how they figured out the problem (usually a less technical approach that others grasp easily). These practices helped to reinforce the learning objectives in both groups. This should not be news to anyone, but there is a delicate balance between catering to specific needs of a few individuals and those who do not require additional attention. Why did I need to ask this question in the first place? I submit to you that targeting technology instruction in the classroom is similar to traditional instruction. I used the same strategy above for both Excel and Mathematica.

Question 2, How much time do you devote to fundamental calculus verses the capabilities found on your laptop? Obviously the focus must be on fundamental calculus. Upon completion of this core course it is imperative for students to take with them basic recall knowledge essential for success in follow-on math and science courses. Why are my convictions so strong? Why did I feel compelled to write this article? I am primarily writing in response to a student's email I got the other day with regards to a future technology homework set. We typically assign such sets and then cover included material prior to the submission date. Here is the email:

-----Original Message-----
From: X
Sent: Tuesday, April 22, 2003 9:06 PM
To: Billie, S. LTC MATH
Subject: Homework set

LTC Billie

Sir I was wondering if the derivative of e^x = e^x? because in my mathmatica I get the same thing as the function.

Respectfully,
CDT PFC X
'06

As you can imagine, I was a little taken aback by his question. Technology is great, but a clear understanding of fundamentals is imperative. Often I feel like the two compete for classroom time. Much like the soldiers transforming to the objective force on the digitized battlefield, we are trying to transform students to the objective core course in the digitized classroom. To lessen student's technology learning curve, I suggest incorporating technology into the beginning of all classes in their first semester. Until these skills are found in core high school curriculums, departments should concentrate their first week's instructions on their primary technology tool (English/History – word processing, Chemistry - Excel®, Mathematics - Excel® and Mathematica®, etc.) This seems like a hefty investment, however, if a proper foundation is laid, then the technology demonstrations throughout the semester will act more as continuity threads as opposed to learning hurdles.

A realistic calculus technology primer should include lessons on both Excel and Mathematica with the instruction focused on basic operations within each software package. Excel lessons should concentrate on basic spreadsheet modeling, graphing capabilities and building embedded formulas. Similarly the Mathematica lessons need to concentrate on basic operations and cover , at a minimum, the concept of cell groupings, common commands, table construction and graphical plots. Class time should be dedicated to specific learning objectives and be self-paced (similar to the Mathematica tutorial offered on the Air Force Academy's Department of Mathematical Science webpage http://www.usafa.af.mil/dfms/mma/intros.htm); out of class assignments would then reinforce skills. Daily interaction and planned continuity threads throughout the course would allow cadets to expand their basic knowledge at a more realistic pace.

As far as Cadet X's email, I directed him towards his calculus text book and told him to be ready to brief why the derivative of e^x was in fact e^x during the next class period. Not to belittle our advancement of technology, but not at the expense of the basics.

I think we risk becoming the best informed society that has ever died of ignorance.--Reuben Blades

Old School vs. New School: A Militaristic Approach

Major Paul L. Goethals, USMA,
Department of Mathematical Sciences

"To be an effective teacher, you must be professionally competent; then you must create conditions in which your subordinates can learn ... In most cases, your people will learn more by performing a skill than they will by watching you do it or by hearing you talk about how to do it ...

There are techniques and methods involved in teaching that have nothing to do with how good you are on the job; you must know both the skills related to the subject and another set of teaching skills."

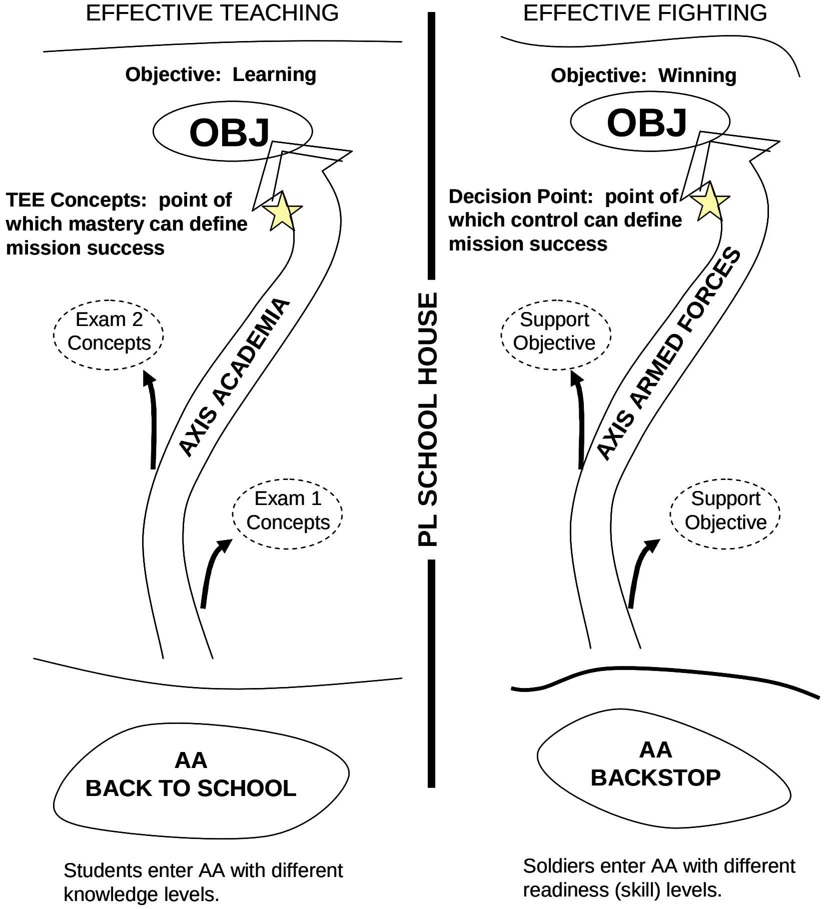
- FM 22-100, Army Leadership, dtd August 1999.

As Platoon Leaders some years ago, we were told time and time again to plan and execute realistic, "hands-on" training in an effort to follow "train as you fight" doctrine and prepare our units for battle. Yet at some point in our careers, we can recall observing a squad's training, seeing the soldiers sitting around a butcher board desperately trying to stay awake, while the Squad Leader or Platoon Sergeant covers the topic of interest for 55-60 minutes. With the attention level of the soldiers quickly gone after the first 15 minutes, many of the important concepts were lost among the squad members. It is not surprising that we tend to lean towards this traditional practice of lecturing for an extended amount of time, as we are all brought up through grade school and high school with this as our only interpretation of teaching style. New methods and practices, however, take a more 'hands-on' approach, incorporating new technology and emphasizing the quality of the instruction versus the quantity. Although there is nothing that shows these new methods result in higher test scores, they do make a large difference in the student's attitude toward learning and their understanding of the concepts, through inspiration and motivation. These new methods have actually made teaching more effective, producing students who are more likely to retain the information, just from their learning experiences.

Prior to each semester, whether we know it or not, we develop a "battle plan" that our students must follow in order to maximize their learning curve. In an effort to ensure these students master the concepts associated with the course (i.e. obtain the objective), we develop a final exam with the desire that mastery of this exam shows mastery of the material. The course is usually then divided into "blocks," in which a student's performance on supporting exams for those "blocks" shows mastery of that particular subject area. And along the way, the students conduct "battle drills", (i.e. quizzes or homework), in an effort to prepare them for the "big fight" – the exam. Each student starts the semester with a basic load of knowledge, or readiness level; some are more ready than others, and fight a three to four-month long battle with a different objective in mind. The main objective of some students is to survive the battle (pass the course) and for others, their objective is to destroy the enemy handily (obtain a high grade). How well these students perform depends heavily on their ability to visualize the battlefield, that is, understand the concepts, as presented to them by their teacher.

"Battlefield visualization is the process whereby the commander develops a clear understanding of his or her current state with relation to the enemy and environment, envisions a desired end state, and then visualizes the sequence of activities that will move his or her force from its current state to the end state .... It is critical to mission accomplishment that commanders have the ability to visualize the battlefield."[1]

The focus of our "battle plan" is effective teaching, through which we aim to maximize the student's learning of the material. Without even knowing it, our plan takes shape:



Now in the modern age, our military relies greatly on its superior technology to provide the best possible picture of the current situation on the battlefield. Everything from the Unmanned Aerial Vehicle (UAV) to the satellite miles and miles above the "fight" - we gain a clear and accurate understanding of our objectives. Without this technology, our ability to visualize the battlefield is hampered; our road to the objective is unclear, and confusion during the fight results.

"Developments in information technology will revolutionize - and indeed have already changed - how nations, organizations, and people interact. The rapid diffusion of information, enabled by technological advances, challenges the relevance of traditional organizational and managerial principles."[2]

In much the same way, new methods in the classroom are centered around the use of technology (computer software, visual demonstrations, etc.) in an effort to enhance the student's learning of key concepts, unlike the traditional classroom practices of using blackboard and chalk to visualize concepts. Laptop computers in the classroom enable students to receive 'hands-on' instruction through new software specifically designed to enhance their learning of the material. "Smart board" technology allows teachers to 'draw' and instantly 'record' their lecture notes for each lesson, so that students can focus more lecture time on discussion of the material versus note-taking. And more and more, universities are moving to the digital classroom, offering entire degree programs online through distance learning opportunities. With the use of technology in the classroom, new teaching methods have a defined advantage over traditional teaching practices, by increasing the student's ability to grasp the necessary concepts in the course.

In our military operations, we depend on quality training during peacetime to prepare us to fight future battles. "Training must be rigorous and as much like combat as is possible while being safe. Hard training is one way of preparing soldiers for the rigors of combat."[3] In the classroom, the best way to prepare students for solving problems on exams is to have them solve problems in class. New methods of teaching involve students spending much of the scheduled class time working designated problems on chalkboards or at their desks on laptop computers. With these new methods, students play a more active role in their learning and leave each lesson with a greater confidence in their ability to solve problems associated with the concepts in the course. Teachers who employ this technique of instruction have a greater understanding of the level of knowledge his or her students have, as errors are seen and corrected on the spot.

"Certain conditions help people learn .... Involve the subordinate in the learning process; make it active. For instance, you would never try to teach someone how to drive a vehicle with classroom instruction alone; you have to get the person behind the wheel. That same approach applies to much more complex tasks; keep the lecture to a minimum and maximize the hands-on time."[4]

Contrary to this new style of teaching, the traditional practice of lecturing to the students for the duration of the class assumes all of the students are at the same knowledge level, and minimizes student-teacher interaction.

In moving to new methods of teaching, there is one strength of traditional teaching practices that must not be overlooked or neglected - challenging the student. Typically, the traditional practices of the lecture puts the burden of work on the student, so that one might argue it provides a greater challenge to the student to learn the concepts. In moving to these new methods of teaching, we may have a tendency to create a complacent student – one who feels so confident that he or she will learn the concepts "in class", that they do little to prepare "outside class". New methods must ensure that students are given problems with higher degrees of difficulty, creating an environment that motivates the students to learn as much as possible. "A unit (staff) constantly needs challenging problems to solve if it's to build the attitude that it can overcome any obstacle ... Great confidence comes from training under conditions more strenuous than they would likely face otherwise."[5]

So if there is no overwhelming evidence in test scores to show that new methods of teaching are more effective than traditional practices, how does one determine that they are actually more effective? In military operations and training, we rely on the After Action Review (AAR) as our feedback mechanism:

"Individuals benefit when the group learns together. Properly conducted, an AAR is a professional discussion of an event, focused on performance standards, that enables people to discover for themselves what happened, why it happened, and how to sustain strengths and improve on weaknesses. With input from the whole team, your people will learn more than if they just think about the experience themselves."[6]

Similarly, in reviewing or judging the effectiveness of teaching methods, we must rely on student feedback. Today's students desire to be inspired - they want to understand the derivation, the associated problems, and most importantly, the application of the concepts we cover in the classroom. By using new technologies and new teaching methods in the classroom, we increase the interest that students have in our specific subject areas, and thus, enhance learning in these areas. In striving to be effective teachers, we should embrace the technology available in the classroom and utilize all methods to the fullest potential.

"The creative leader is one who will rewrite doctrine, employ new weapons systems, develop new tactics and who pushes the state of the art."

- John O. Marsh, Jr.
Former Secretary of the Army

References

[1] FM 101-5. *Staff Organizations and Operations*. Headquarters, Department of the Army, Washington, D.C., 31 May 1997, p. 1-3.

[2] FM 100-6. *Information Operations*. Headquarters, Department of the Army, Washington, D.C., 27 Aug 1996, p. 1-2.

[3] FM 22-100. *Army Leadership*. Headquarters, Department of the Army, Washington, D.C., 31 Aug 1999, p. 3-4.

[4] FM 22-100. *Army Leadership*. p. 5-26.

[5] FM 22-100. *Army Leadership*, p. 6-22.

[6] FM 22-100. *Army Leadership*, p. 5-26.

Old School vs. New School: The Use of Technology in the Calculus Sequence

Dr. James S. Rolf and Dr. Michael A. Brilleslyper, USAFA, Department of Mathematical Sciences

"We seem to be at some sort of critical point, but what type? Is it a local or absolute minimum or perhaps an inflection point? Everything just looks flat—perhaps we need to zoom out and get a better perspective on the whole thing."

If the above discussion only concerned the analysis of the graph of a function, then we would not have much to write about. Instead, it is a statement about our efforts to reform first year calculus here at USAFA in an attempt to thoroughly and effectively integrate technology. Over the past year we have been entrenched in the implementation of our "new" calculus. We are at an educational critical point and it is time to step back and take a hard look at where we have been and where we believe we are going.

The importance of calculus in the curriculum has not diminished. As the future demand for engineers and scientists continues to rise, the need for students with a solid grasp of key calculus concepts will increase. However, the need for lengthy algebraic hand calculations and the ability to sketch intricate geometric drawings has been replaced by the need to model ill-posed "real world" problems and to apply today's advanced technological tools to solve them. The fundamental question for mathematics educators is can you achieve advanced problem-solving skills without first developing basic mechanical skills?

We believe the answer is yes, though not without much pain and suffering both for faculty and students. The Mathematical Sciences Department at USAFA has just completed the first year of a reform effort. There have been successes, frustrations and numerous surprises along the way. In the remainder of this brief article we will discuss student and faculty reaction to our efforts, as well as giving an overview of our methods and curriculum. There will be no definitive answers to questions we have raised as this represents the beginning of a discussion, not the end.

When we began to develop our new course, our vision was a dynamic mathematics classroom where students utilized a variety of technological tools to explore and solve challenging problems. We hoped for an environment in which students enthusiastically embraced the tools of their generation, moving easily between spreadsheets, Java applets, and computer algebra systems. We saw a course structure that encouraged deep conceptual reasoning and one that required students to write articulately about mathematics. We pictured a curriculum that made extensive use of visualization and advanced graphics—one that would certainly appeal to the fast-paced MTV generation. We expected students to embrace the laptop computer whole-heartedly and to demonstrate technological wizardry that would soon leave their older and out-of-touch professors far behind. In short, we set out to design the calculus course for the 21st century. We included all the needed components: interdisciplinary projects, short conceptual writing assignments, online basic skills exams, midterm and final exams that utilized laptop computers, and daily online questions concerning required reading. We were confident we had designed a good course. We held firm to the notion *"If we build it they will come."*

Well, our first year is over and it seems that our 21st century students prefer the mathematical methods of the 19th century. Why? The following quotes obtained through numerous forms of assessment may lend some insight. We offer our own brief analysis of what the quotes may actually mean. Note that though we have chosen quotes by individual students, they are representative of the feelings of a large number of students in the course.

- *"Start giving us traditional calculus problems out of the textbook worked with pencil, paper, & calculator." "Start using real calculus - teach math not fuzzy math."* These students clearly perceive that what we are doing is "not real math." They believe that "real math", or "traditional math", is something done with a pencil, paper, and calculator. Never mind that traditional calculus has been done for 300 years without a calculator!

- *"Stop having so much conceptual math." "Less story problems, more bookwork with numbers."* These students are objecting to our emphasis on higher order learning skills (analyzing, synthesizing, and interpreting results) and our emphasis on conceptual understanding. They are typical of the cadet who requests an exam of twenty derivatives.

- *"Stop using English for explaining stuff. That's why it is called math, to use formulas." "I want Math, not Eng 111."* These students do not understand the value of being able to articulate in writing mathematical ideas. They, and many others, have a compartmentalized perception of the courses that they take and believe that writing should be relegated to the English department. They clearly do not grasp the correlation between the ability to write about a concept and how much one understands that concept.

- "*Stop assigning projects that have more to do with economics than they do with math.*" This complaint is in reference to an optimization project in an economics context. Again, this student has a compartmentalized perception of learning. He/she believes mathematics should only be done in the math department and economics should only be done in economics department. This comment is also indicative of the fact that this student (and many others) does not value our paradigm shift from an emphasis on calculations to an emphasis on conceptual understanding and the ability to model "real world" problems.

- *"Start teaching! Some things that were on the test weren't ever practiced." "Not so much emphasis on theory, no one really cares."* These students are indicating that they want us to teach by giving examples just like assigned homework problems and then give tests just like assigned homework problems. They have trouble with our requirement of developing higher order thinking skills.

In summary, the students are complaining that we have 'changed the rules' and are emphasizing different things (i.e. higher order thinking skills). We do not place as much value on hand calculations and are placing much heavier emphasis on understanding and applying mathematical ideas. Since we are not meeting student expectations concerning the way in which they should learn mathematics, they are uncomfortable with our changes.

The general student reaction to our changes has certainly been one of the biggest surprises we have faced. When we set out to integrate technology in our courses, we believed that this internet-savvy generation of students would embrace our different approach and feel much more comfortable with the use of technology than our instructors would feel. Our experience shows that the opposite is true—the students have vociferously complained and faculty have largely embraced our new changes.

The recent history of mathematics education has seen numerous successful revolutions. Slide rules replaced hand calculations, scientific calculators replaced slide rules, graphing calculators replaced scientific calculators, and now laptops with multiple software tools replace graphing calculators. Each of these transitions was viewed with great suspicion and skepticism by members of the profession. Like each of the others, the transition to using laptop computers will not be without some strife. However, in our case it seems to be coming more from the students than from the Ivory Tower. In retrospect, we should not have been surprised at student response since we are changing the 'rules of the game' and are expecting the students to engage in different kinds of learning than they are used to.

Our Objectives

Since the mission of our institution is to produce Second Lieutenants for the Air Force, we saw our primary role in this mission as one that would develop the problem solving skills of our students in a way that would best prepare them for their future careers. This mission has certain far-reaching implications.

Our mission suggests that we need to require a deeper kind of understanding behind mathematical concepts so that students can learn how to apply knowledge in varied contexts. Students should also be able to handle some ill-posed "real world" problems by building models and interpreting consequent results. They should to be able to communicate clearly the technical results from any models that they build. The importance of this last point was not fully realized until we had begun to implement our ideas. We have come to realize that since technology can perform many standard calculations, the need for students to articulate *why* they are doing what they are doing and *what* the calculation means has become a crucial component of our new course. Finally, our mission also suggests that we would need to prepare students to work in teams since they would be doing this through the duration of their career, whether in the Air Force or not. Currently, this happens when students work in teams on more difficult inter-disciplinary projects.

As we set out to construct courses that would integrate technology in a way that would reflect these objectives, we quickly realized that would have to consider the consequences in three areas: the content of our courses, i.e. the curriculum, how we do business in the classroom (our pedagogy), and how we understand what our students have learned (our assessment). Technology is available to both the students at all times. These tools have opened up many nice venues for helping students to "discover" important mathematical ideas. The Air Force has a truism that we have adopted in our assessment— "Train Like You Fight." So we allow all technology that a student uses on a daily basis to be used on almost all of our assessment instruments.[1]

Conclusion

The intense and somewhat hostile reactions of many of our students indicate that we are at a critical point. Currently, faculty view this critical point through a much different lens than the lens students are using. Still, we are committed to our dual objective of providing our students with the finest education possible and preparing the best Second Lieutenants for the United States Air Force.

Our commitment to these interrelated objectives means that we must be committed to changing the student culture in order to do a better job of developing problem solving skills. This is an uphill battle; student reaction as been swift and strong. But we believe that over time the student culture will improve and the critical point that we now face will become a (hopefully) distant memory.

Wreckless Abandon[2]

Prof. Brian Winkel, USMA, Department of Mathematical Sciences

One measure of teaching effectiveness is "teacher liberation." In

[1] The once exception to this policy occurs when we assess the Fundamental Skills of a students. We allow no technological aids during online exams in which we examine the differentiation and anti-differentiation skills.

[2] We note that rather than use Reckless Abandon we prefer Wreckless Abandon, because that is the hope -- teaching with reckless abandon, letting technology carry the day and pick up the pieces of tedium called algebra and symbol manipulation, but without "wrecks."

many evaluations it is teacher enthusiasm that marks student interest and if the teacher feels uninhibited, energized, and inspired, then so too will the students. A second measure of teaching effectiveness is how emboldened the students are to explore, to learn on their own. We offer some illustrations of how using technology to teach with "wreckless abandon" can enhance both the teacher's and the students' measures of effective teaching and effective learning. We refer to the Old School here as without technology and the New School with technology.

Long ago I was given a gift in my teaching career – a VAX Workstation and Maple computer algebra software. This was a precious gift and I was on my own to explore how to use it in my teaching. It could be abused, e.g., assign students "by-hand" work and use Maple to check the answers. I was determined to do more creative things with the software, to go places I had not been able to visit, but I was not quite ready to proceed with "wreckless abandon."

One of the first things I did was to reverse the game on assembling information on intercepts, derivatives, slopes, etc. to gain information on a function in order to render a plot of the function. I simply used Maple to plot the function FIRST and then we "studied" it like one would study a creature. We analyzed it with derivatives to try to explain why it went the way it did. We sought its peaks, its troughs. We had the object in front of us and we were strengthening our understanding of the calculus notions by this scrutiny. This is a different approach than using the calculus information to build a graph, the latter could now easily be done by Maple, and so this freed us to study the function with the power of Maple and ask deeper questions about the function and its properties.

For example, we could plot and see the first and second derivative instantly along with a plot of the function itself and thus match up properties of derivatives with behaviors of plots instantly. We could ask students to identify plots of $f(x)$, $f'(x)$, and $f''(x)$; indeed we had students make up quizzes for each other doing just that, namely - for a plot of $f(x)$, $f'(x)$, and $f''(x)$, identify each. Old school could not do all this because the technology was not there. However, some strengths of the old school were inquiry, students' taking ownership for learning, and constructivism in learning. Technology enabled all of these nicely.

A weakness in the old school was the universal expectation of flawless algebraic skills, of attention to the minute details of symbol manipulation, and of the development of a set of clever tricks to reduce problems or equations to some special form so another trick could be applied. Now with computers performing the tricks, the student is free to move beyond manipulation and recalled tricks to see the big picture without the hindrance of algebra errors, lack of insight into the domain of tricks, and pages of hand written calculations.

Students routinely question their work more with the new school of technology, for to find an error is then but to make a simple change in a term and re-execute the calculations of a page or more. While in the old school, before technology, to find an error meant redoing all the hand calculations at best, or throwing that write-up out and starting anew. Most students did not redo their work if they found an error. Most students did not even go looking for an error because they understood the tremendous consequences – better to just hand it in and hope for the best. New school technology changes all that for the better.

Fourier Series

We give two examples of how the new school thinking with technology actually enabled us to teach with wreckless abandon. The first is Fourier series. For a number of years now we have introduced students to Fourier series, by not introducing them to Fourier series, but rather having them discover how to approximate a function with a "bunch of sinusoidal generators." They never fail to discover the theory, to make it their own, and to enjoy it. It is the technology that permits this discovery and the reinforcing feedback to support the students as they conjecture and move ahead.

We introduce the idea of using sine functions to approximate functions, usually over the interval [-π, π]. We have been very successful in getting students to discover Fourier series – we never use that name, we stick to the idea of building signals with signal generators. We use signal generators $g_n(x) = \sin(n\pi x)$, $n = 1, 2, 3, 4, \ldots$ to generate signals $f(x)$ --- specifically odd functions in the interval [-π, π] --- as this is a reasonable task for new school students equipped with technology, but not as notationally confusing as the full general Fourier series.

$$\sum_{n=1}^{m} a_n \cdot \sin(n\pi x)$$

We ask the students to find constants a_n such that the sum is a good approximation of $f(x)$. i.e.

$$f_{approx}(x) \approx \sum_{n=1}^{m} a_n \cdot \sin(n\pi x)$$

Students discover the least square criteria – they do so EVERY time on their own and then they set out to try to minimize the least square error. Here l = π.

$$\int_{-1}^{1} \left(f(x) - \sum_{n=1}^{\infty} a_n \cdot \sin(n\pi x) \right)^2 dx$$

They first see if they can do this for a finite approximating sum of sine functions, $n = 2$.

$$\int_{-1}^{1} \left(f(x) - \sum_{n=1}^{2} a_n \cdot \sin(n\pi x) \right)^2 dx$$

We use a nice function, say $f(x) = x$.

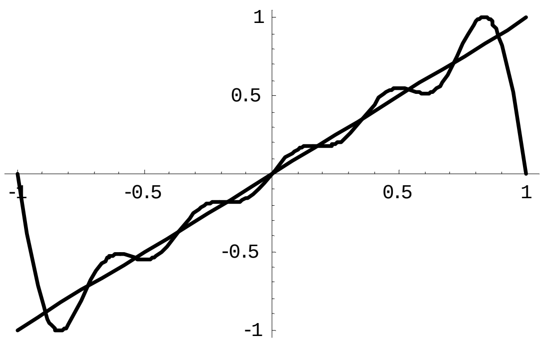
This sum of squares is a function of two variables, $a = a_1$ and $b = a_2$, and we can minimize it using calculus, computing the partial derivatives with respect to a and b and setting these derivatives equal to 0 to determine the a and b which make $S(a,b)$ minimum.

$$S(a,b) = \int_{-1}^{1} \left(x - a \cdot \sin(\pi x) - b \cdot \sin(2\pi x) \right)^2 dx$$

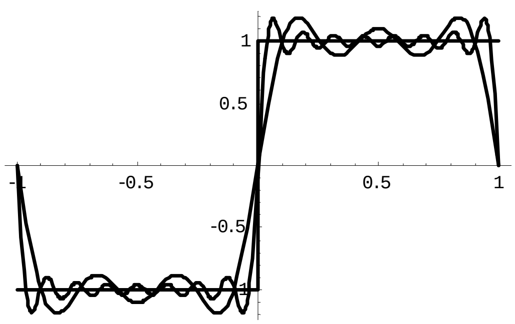
Then we do this for three terms $S(a,b,c)$, and four terms, $S(a,b,c,d)$. We observe a pattern and predict what the a_n's will be at the same time we confirm our bettering of the approximations with plots. This is an example of immediate feedback with new school technology.

Now we generalize with *2, 3, 4*, etc. terms using general odd functions $f(x)$ to obtain the a_n's.

We confirm our strategy with plots for *f(x)* = *x*; *n* = *5* here.



Here is a plot of *n* = *3* and *n* = *5* term series over a step function.



As you can see from this stream of consciousness development the students can discover the concept of a Fourier series with the new school of technology tools of differentiation and plotting in a computer algebra system.

Ecological modeling –more and more realistic efforts.

In most modeling efforts students are encouraged to build a classical predator prey-model of the following form:

$$N'(t) = r \cdot N(t) - N(t) - a \cdot N(t) \cdot P(t)$$

and $P'(t) = -c \cdot P(t) + b \cdot N(t) \cdot P(t)$.

We encourage students to build such a model for *N'(t)* =____ and *P'(t)* = ____ stating reasonable assumptions about a closed system predation model with *N(t)*, the biomass of prey, and *P(t)*, the biomass of predators at time t, and to immediately plot the numerical solutions to the system of nonlinear differential equations with a reasoned set of parameters. For example, consider the relationship between *a* (predation notion) and *b* (consumption notion). Thus we see *b* < *a*, i.e. conservation is in effect. The emphasis is on assumptions, on units, on terms, and on interactions between variables, NOT on algebra or calculus solution strategies. It is time for something new in a mathematics class – play! We study the system, its flow, its stability, its range, etc. We see how these are altered by changing parameters, one at a time as a good scientist does in a controlled experiment. We do a reality check as we go, storing up the good points and looking out for the bad ones.

We proceed to find inadequacies in the model that make it unrealistic and attempt to patch it up with an improved model. Students point out the "foolishness" of the *r * N(t)* term in *N'(t)* as it represents exponential or unlimited growth so we attempt to remedy that with the introduction of a carrying capacity and notions from the logistic equation. We continue to keep an eye on the reality check aspect of modeling by examining the consequences – usually with the new school technology. Again we are always checking units, looking at plots.

$$N'(t) = \frac{r \cdot N(t) \cdot (K - N(t))}{K} - a \cdot N(t) \cdot P(t)$$

and $P'(t) = -c \cdot P(t) + b \cdot N(t) \cdot P(t)$

One of the last modifications is to examine the predation term *a * N(t) * P(t)* by thinking of the predator (*P(t)*) as the student and the prey (*N(t)*) as pizza. If *P(t) = 1*, i.e. YOU and *a = 0.1*, say, then if *N(t)* pizzas show up at your door you eat *a * N(t) * P(t) = 0.1*N(t)*1* pizzas per hour --- say *t* is in hours (i.e. one-tenth of ALL the pizzas that show up at your door per unit time!), in particular if *N(t) = 1* you eat 0.1 pizza per hour; if *N(t) = 100* you eat 10 pizzas per hour; if *N(t) = 1000* you eat 100 pizzas per hour; etc. Not so!!! You become full, indeed, you become stuffed. There is a satiation going on here and you can only eat so much per hour so this predation term, *a * N(t) * P(t)* , and the corresponding assimilation term, *b* N(t) * P(t)* , have to be modified to reflect some limiting factor, i.e., as *N(t)* gets larger and larger we can eat only a maximum amount per unit time. We actually devote a whole class period to this one term, with students attempting to model satiation with lots of conjectures, either qualitatively by hand or by implementing them on our technology.

Students come to grips with the fact that they now struggle to offer up a term or modification to the predation term to reflect satiation. They usually can arrive at something like this:

$$N'(t) = \frac{r \cdot N(t) \cdot (K - N(t))}{K} - \frac{a \cdot N(t) \cdot P(t)}{L + N(t)}$$

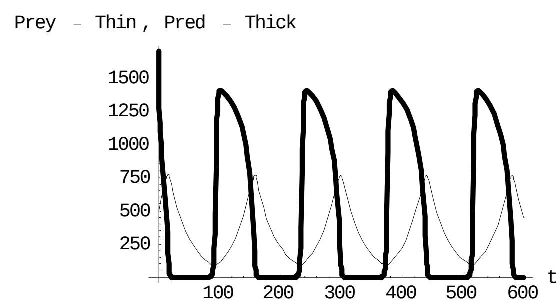
and

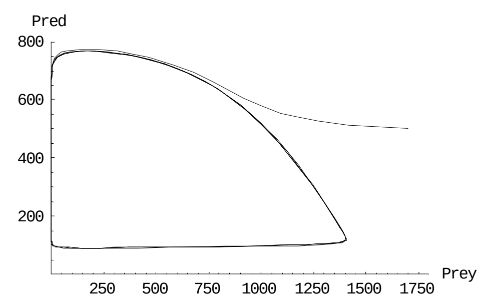
$$P'(t) = -c \cdot P(t) + \frac{b \cdot N(t) \cdot P(t)}{L + N(t)}$$

where *L* is the level of the prey at which the predator feeds at one half its maximum feeding rate, that maximum level being *b*P(t)* and when *N(t) = L* we have a feeding rate of

$$\frac{b \cdot N(t) \cdot P(t)}{L + N(t)} = \frac{b \cdot L \cdot P(t)}{L + L} = \frac{b}{2} P(t) \; .$$

Below we offer two plots of the result of the students' modeling, just some of many plots made on the road to success, only because our new school technology lets us explore, to play "what if" games, and to try out our theories easily.





Conclusion

We have demonstrated several instances of how the new school with technology is richer than the old school of pencil and paper, of how the students explore in this new world, and of how we teach to a new school of technology-enabled students. Liberate yourself from the tedium of by hand algebra and symbol manipulation, take risks, play, improvise, build models, do "what if" games, experiment, let your students lead, and enjoy the new school approach with technology.