MATHEMATICA MILITARIS

THE BULLETIN OF THE
MATHEMATICAL SCIENCES DEPARTMENTS
OF THE FEDERAL SERVICE ACADEMIES

TECHNOLOGY, TEACHING, AND FUNDAMENTAL SKILLS

Volume 16, Issue 1
Spring 2006

EDITOR'S NOTES

Happy (belated) New Year! Apologies to those whose January was blighted by the absence of the latest *Mathematica Militaris*. The Editorial Team hopes that the current collection of articles proves well worth the wait.

The theme of the present volume is "Technology, Teaching, and Fundamental Skills," with four thoughtful articles on this theme, yet with very different topics!

One recurring question in planning mathematics curricula today is: what counts as a "fundamental skill" in the era of computer algebra software? James Rolf, Michael Brilleslyper, and Andrew Richardson recount how the United States Air Force Academy (USAFA) has addressed this question in teaching core mathematics courses, and detail the use of online software to assess student progress.

LTC Mike Huber of the United States Military Academy (USMA) follows up with thoughts on fundamental skills, computer algebra systems and Hamming's dictum, "the purpose of computation is insight, not numbers."

In the third article, Dr. Brad Warner and Lt. Col. Lem Myers next describe an ongoing effort to provide an alternative to instructor-centered courses at USAFA, using a modeling and team problem-solving approach, and detail the advantages and pitfalls of their first attempt. Their paper should be of great value to anyone contemplating a similar approach.

The final article, submitted by your editor in chief, recalls an interesting final project problem from a class on the mathematics of Einstein's General Theory of Relativity given in the Spring 2005 term at USMA. This class was given in response to cadet requests (!) and the final project required mastery of manual computation and yet, would have been impractical without use of the computer algebra system *Mathematica.*

I hope you enjoy the present rendition and that you will become inspired to share your own ideas, techniques, and strategies with your colleagues in future volumes.

Be sure to visit our website for past issues: *http://www.dean.usma.edu/math/pubs/mathmil/* .

CONTENTS

SUBSCRIPTIONS TO *MATHEMATICA MILITARIS*:

If you would like to be on our mailing list, please send your name, address, and affiliation to:

Editor, *Mathematica Militaris*
Department of Mathematical Sciences
United States Military Academy
ATTN: MADN-MATH
West Point, New York 10996

Teaching Fundamental Skills at the United States Air Force Academy

James S Rolf, Michael A. Brilleslyper, and Andrew X. Richardson
Department of Mathematical Sciences, United States Air Force Academy

I. Introduction

MATHEMATICS courses form the cornerstone for a scientific education. At most colleges and universities the calculus sequence plays a critical role in meeting the needs of other disciplines and future mathematics courses. Yet calculus education has remained far from static. The past fifteen years have seen dramatic changes in the content of calculus courses, the way in which they are taught, and how students are assessed. Advances in technology and the calculus reform movement have had a tremendous impact on the calculus sequence. Throughout these changes, there has been a constant and on-going debate over the various roles of rigor, mechanical skills, mathematical modeling, and the appropriate use of technology. Very little has been settled, and it is easy to find a wide spectrum of approaches in mathematics education. These approaches vary from institutions that teach calculus the way it was taught in the 1960's to departments that use computer algebra systems with little regard for manipulative skills.

In the mathematics department at the United States Air Force Academy, we have implemented a course sequence that balances mechanical skills with the overarching goal of developing good critical thinkers who grasp the key conceptual ideas of calculus. Over several semesters we have developed a curriculum that is now being used across our technical core courses and is affecting more than 1100 students per semester. All of our calculus courses are constructed around several components, each supporting the main goal of developing good problem solvers. Our assessment instruments include Fundamental Skills Exams (FSEs), conceptual/problem-solving exams, short writing assignments, and group projects that emphasize modeling. In all of these (with the exception of FSEs), we believe that an integrated and meaningful use of technology for visualization, exploration, and computation will best facilitate deeper conceptual understanding along with increased modeling skills.

For many departments, the push towards a more conceptually based modeling curriculum comes at the cost of diminished mechanical skills. For many faculty members this can be a hard pill to swallow—we simply expect our students to be proficient at computing derivatives and anti-derivatives. In fact, many people believe that a solid grasp of calculus is only possible after mastering the algebraic manipulations of computing derivatives and integrals. We do not ascribe to this point of view.

We believe that the development of critical thinking skills is a non-linear process. Some students may be able to think critically because they understand well the important conceptual ideas in place behind certain mathematical objects. These same students might also be less skilled in successfully completing detailed computations. Other students may be very proficient in memorizing facts and procedures and after some period of time, develop important conceptual understanding that allows them to think critically. Other students develop problem solving abilities by working in groups. In short, we believe the path to developing the ability to analyze, synthesize, and evaluate is a difficult and varied path. And because this path is a difficult one, we must spend significant time in the classroom focusing on these difficult skills. But at the same time, we believe that a low proficiency in the algebraic manipulations of computing derivatives and integrals will ultimately limit the ability of students to model and solve problems. So while we must spend the majority of class time helping students developing higher order thinking skills, we cannot ignore the need to help students learn basic skills.

Thus, with this framework in mind, the remainder of this paper will focus on how we utilize Fundamental Skills Exams (FSEs) to develop these basic skills.

II. Fundamental Skills Exams

We believe that high proficiency in mechanical skills comes through memorizing a basic set of rules and then practicing many problems that develop the pattern-recognition skills that are necessary to correctly apply these rules. One could certainly devote an extensive amount of class time practicing, for example, various combinations of the power rule for computing derivatives, the chain rule, the product rule, etc. However, we believe that a minimal amount of class time is all that is needed and that the majority of practice is best done outside of the classroom.

In order to provide incentives for students to spend time practicing these skills, we have devised a series of Fundamental Skills Exams (FSEs) that are given in the core technical courses in our department.[1] The availability of robust online testing software[2] enables a student to practice an unlimited number of problems prior to taking an FSE. We further utilize this testing software to give students the official FSE in a proctored environment. These exams are graded on a pass/fail basis and students must demonstrate a high proficiency in order to pass. Currently our standard for passing is 80% proficiency, but we hope to move towards a 90% standard in the future. Additionally, we have imposed a gateway constraint on each FSE. That is, if a student fails an FSE (after being given multiple opportunities to pass at the required proficiency), he/she may not advance to the next course in the mathematics sequence. In short, if a student fails an FSE, the student will fail the course regardless of his or her course average.[3]

The requirement of demonstrating a high proficiency combined with the consequences of failure generally provides plenty of incentive for students. These requirements also impact how we design and administer FSEs. First, we must carefully consider what kinds of questions we ask. We have decided that the content of our FSE's will focus on manipulative skills that require primarily symbol manipulation and basic pattern recognition. In designing our FSEs, we asked ourselves "am I willing to fail a student if he/she is not able to demonstrate this particular skill?" This means that we usually limit the number of steps involved in solving an FSE question to two or three steps. Secondly, since the stakes of an FSE are high for the student, we give each student multiple attempts to pass each exam. This semester, we gave each student four attempts to pass an FSE at 80% proficiency. We have provided students two incentives to pass this exam early.

- Each exam is worth between 5% and 10% of the course points.
- The amount of credit a student receives for passing an FSE decreases with each attempt.

Attempt on which FSE is passed	#1	#2	#3
Grade	100%	80%	60%

Table 1: Grade vs. attempt on FSEs.

[1] Our technical core includes Calculus 1, 2, and 3, Differential Equations, and Engineering Mathematics. FSEs are currently administered in Calculus 1, 2 and 3, with plans to implement in the other courses beginning Fall of 2006.

[2] We currently use WebAssign for this purpose. See http://webassign.com for more information.

[3] Since this is the first year of using this policy, we have agreed to examine each potential course failure due to failure of an FSE on a case-by-case basis. So in practice, our policy is "you are at a serious risk of course failure if you fail an FSE." But our intent is to make course failure an automatic consequence.

Observe that a student receives no points for passing on the last attempt other than removal from the category of "automatic course failure due to failure of an FSE." This fourth attempt is given on the last lesson of the semester in an attempt to prevent students from giving up in the course. The first attempt is administered during part of a class period. Subsequent attempts are given in a proctored environment outside of the classroom, with approximately 25-30 allotted for attempt.

After spending minimal class time demonstrating basic mechanical skills problems we provide students with an extensive list of potential problems using online testing software. The software allows coded problems to be algorithmically generated which allows for randomization of problems. Thus, one student may encounter the problem of finding the derivative of sin(2x), while another student may get the similar problem of taking the derivative of sin(3x). Each of these questions involves exactly the same thought processes. We further categorize each practice problem as "easy," "medium," or "hard." The official FSE then draws a pre-specified number of questions from each of these categories and randomizes the order of questions for each student. The ability to practice the needed skills allows students to tailor the amount of work they do to their own needs. Weaker students practice more and stronger students need only focus on perhaps a few types of questions.

III. Multi-course implementation

The use of FSEs allows us to emphasize critical material across several courses. For example, in calculus 1, students learn how to take derivatives and must pass a derivative FSE. The same exam is then given at the beginning of calculus 2 in order to emphasize the importance of the topic and to show the relevance of past material to the current course. A second FSE covering anti-derivatives is given later in the same semester of calculus 2. Table 2 shows the topics covered on each of the FSEs in our calculus sequence.

Calculus 1	FSE 1: Functions, algebra, and trigonometry
	FSE 2: Derivatives
Calculus 2	FSE 1: Derivatives
	FSE 2: Anti-derivatives
Calculus 3	FSE 1: Single variable derivatives and anti-derivatives
	FSE 2: Partial derivatives, anti-derivatives, and vector properties

Table 2. FSEs across multiple courses.

Note that the FSE structure naturally lends itself to spiraling between new and prerequisite material over several courses. We believe that this "enforced spiraling" will greatly help student retention of fundamental skills. Beginning in the fall of 2006, we intend to expand our FSE program to include all of our technical core courses by adding Differential Equations and Engineering Mathematics along with probability and statistics courses.

IV. FSE Design

Typical questions on FSEs are process-oriented as opposed to conceptual questions. They usually involve brief (i.e. one or two steps) calculations with pencil-and-paper. Students enter their answers into the testing software. At the end of the exam, the software evaluates each answer as "correct" or "incorrect." We do not ask multiple choice questions nor do we give partial credit for incorrect answers.

We provide several examples of the kinds of questions that are on our FSEs. Each of the following examples is taken from the

MEDIUM category along with a brief statement of how that designation was determined.

- Algebra FSE
  1. Simplify the following expression: $\left(\dfrac{2x^{-1}y^2}{8xy^3}\right)^{-3}$.
  2. Combine into a single fraction in lowest terms: $\dfrac{2-x}{x+3}-\dfrac{2}{x-3}$.

Both of these questions focus on critical algebra skills. Lack of algebra skills often affects a student's ability to be successful in higher mathematics courses. Our algebra FSE attempts to hold students accountable for some basic algebra proficiency, while not devoting much time in class to this prerequisite material.

- Derivative FSE
  1. Find the derivative of $f(x)=\cos^2(2x)$.
  2. Find the derivative of $g(x)=\cos(2x^2)$.

Both of these problems require some proficiency with algebra (order of operations) and the use of the chain rule. We expect all calculus students to master these types of questions.

- Integral FSE.
  1. Find the antiderivative of $f(x)=4x-\dfrac{1}{2x}$.
  2. Find the antiderivative of $g(x)=x\sin(3x^2)$.

The first problem commonly results in an algebra error of keeping the 2 attached to the x. The second is a standard substitution problem that we expect all our students to be able to do.

V. Results

We first implemented Fundamental Skills Exams in a widespread manner here at USAFA in Calculus 1 and 2 during the fall of 2005. Our results from the fall 2005 semester are summarized in Tables 3—7. The percentages in the table indicate the cumulative pass rate for each FSE in each course. Note that the fourth attempt on FSEs had not yet occurred when this paper was written.

In Calculus 1, students tended to not perform as well on the first attempt of FSE 1 as they did on the first attempt of FSE 2. But in both cases, the same cumulative pass rate was attained by the third attempt of each FSE.

	Attempt 1	Attempt 2	Attempt 3
FSE 1: Functions, algebra, and trig	43%	85%	94%
FSE 2: Derivatives	91%	95%	94%

Table 3. Calculus 1 FSEs.

In Calculus 2, students initially struggled on both derivative and integral FSEs , but eventually reached a cumulative pass rate of around 90%. We anticipate a stronger performance on the derivative FSE in calculus 2 in future semesters due to the fact that students will be taking this FSE exam for the second semester and will be forced to study this material for a second time.

	Attempt 1	Attempt 2	Attempt 3
FSE 1: Derivatives	33%	75%	89%
FSE 2: Integrals	46%	74%	93%

Table 4. Calculus 2 FSEs.

In Calculus 3, students started reasonably well and finished strong on the derivative/integral FSE.

	Attempt 1	Attempt 2	Attempt 3
FSE 1: Derivative and Integrals	69%	89%	92%
FSE 2: Vector properties, partial derivatives and anti-derivatives	N/A	N/A	N/A

Table 5. Calculus 3 FSEs.

V. Conclusion

We believe that the use of Fundamental Skills Exams in our technical core courses holds much promise. It puts the onus of learning mechanical skills where it should be—out of the classroom and into the hands of students. This has some important implications. First, it frees up time in the classroom so that we can focus on the (arguably) more important higher order thinking skills such as modeling, synthesizing, etc. Secondly, it impacts how we design in-class exams. Since we do not have to assess mechanical skills, we can ask more in-depth and involved questions. We can ask students to spend time on the exam creating mathematical models. And since we are not testing student efficacy in memorizing basic skills, we allow students to use technology (i.e. computer algebra systems, applets, etc.) during the in-class exams. This further enhances our ability to assess student ability to solve complex problems. Thirdly, because we are requiring students to demonstrate high proficiency on fundamental skills across multiple courses, we believe that students will ultimately internalize these basic skills and be able to recall them as needed in related courses.

Ultimately, we see the Fundamental Skills Exam as a great tool that not only forces students to learn basic skills but supports our goal of developing students' ability to think critically.

Fundamental + Skills = Success in Solving Problems

LTC Mike Huber, USMA, Department of Mathematical Sciences

What is a "fundamental skill" of mathematics in the age of laptops and computer algebra systems? Ask a group of freshmen college students, university faculty members, or businessmen in New York City for their views on fundamental mathematics skills and laptops and you will get different responses. Better yet, walk into a classroom or scientific research facility and ask folks to raise their hand if they can define or agree upon what a fundamental mathematics skill is. Not a trivial matter. Finally, type "laptop fundamental skills" into Google and see what comes up. You might be surprised. The first entry returned is "Required Fundamental Skills for Students Entering MA104." The top result of about 478,000 (close to half a million) brings a listing from the Department of Mathematical Sciences at the United States Military Academy. (Go ahead, try it!) Agreement on specific answers might be difficult. However, in a general sense, there is a positive movement sweeping the nation about incorporating the use of technology into the mathematics curriculum, to assist with learning fundamental mathematics skills.

According to Princeton University's online dictionary WordNet, *fundamental* is an adjective defined as "serving as an essential component" or "being or involving basic facts or principles." In addition, skill is a noun which is defined as the "ability to produce solutions in some problem domain." When combining this adjective with the noun, we look for an ability to *solve problems* using essential principles. Fundamental skills in mathematics should build a foundation for the student which will assist in solving problems for the rest of that person's life. Sure, each person will continue to learn, but certain fundamental skills will stay with him or her forever. How does the laptop computer figure into that scenario?

Mathematics, science, and engineering education experts have been debating the role of fundamental mathematics skills in the learning process of students for a long time. How often do we hear that our students are lacking good algebra skills? On one hand, many believe that mastering algebra is a fundamental skill that should be taught by mathematics departments and not with a computer. In fact, Leonhard Euler, in the beginning of his preface to *Introductio in Analysin Infinitorum* in 1748 (long before computers were available), wrote, "Often I have considered the fact that most of the difficulties which block the progress of students trying to learn…stem from this: that although they understand little of ordinary algebra, still they attempt this more subtle art." By mastering the early mathematical topics, such as algebra, those in the pursuit of science will succeed in understanding it. Winfred Ernest Garrison supported this idea, writing, "All in good time there will come a climax which will lift one to the heights, but first a foundation must be laid, broad, deep, and solid." I would venture to guess that most, if not all, mathematicians would agree with Garrison's and Euler's sentiments. However, I feel that using a laptop computer in the mathematics classroom can greatly enhance the ability to build that foundation and lift students to the heights when solving problems.

In 2000, the National Research Council's Commission on Behavioral and Social Sciences and Education issued a volume entitled, How People Learn: Brain, Mind, Experience, and School. In it, we find the authors' recommendation that students in mathematics classes should learn how to understand mathematics– a goal that is commonly accepted by almost everyone in the current debate over the role of computational skills in mathematics classrooms. Is the use of a laptop computer in the classroom replacing our fundamental knowledge base? Most mathematicians "see computation as merely a tool in the real stuff of mathematics, which includes problem solving [there it is again], and characterizing and understanding structure and patterns." Many advocates of eliminating computer algebra skills in building the fundamental knowledge base seem to think that the students are dependent on the computer for these skills. They argue that the computer is doing the thinking for the student. Most of us are familiar with Richard W. Hamming's famous quotation, "The purpose of computation is insight, not numbers." By using a computer algebra system (CAS) in a mathematics classroom, I hope we are trying to assist the students in creating insight, visualization, and understanding of the problem to be solved. Hamming's adage suggests that we use the CAS to gain an appreciation and an understanding of the problem (not an understanding of the algebra!) and a possible method to solve it. For example, suppose we want to teach the students about derivatives and their relationships to functions. Going back to the "laptop fundamental skills" link listed above, under #17 of the Required Fundamental Skills for Students Entering MA104, students should be able to understand the derivative as an instantaneous rate of change and the slope of a curve at a point. What better way to understand this than to plot the function and its derivative? A laptop with a CAS can do that during class. The CAS is not taking the place of the student's knowledge (someone has to develop and type in the correct equations), but it can offer insight into how a function changes instantaneously at a point. Syntax, you argue, will be the death of the student.

Maybe, but didn't the student learn syntax in grade school, when he or she was taught order of operations (multiplication before addition, etc.), and don't students learn syntax of sentence structures in language classes?

This brings me to the focal point. Mathematics should be taught in order to solve a problem. Fundamental skills are needed to solve problems. Surely, there are those of us who appreciate mathematics for the theory, for its own sake, and proving a theorem validates our applied efforts. Researchers use mathematics to gain some understanding of the physical world. In his Center for Faculty Development address to the Department of Mathematical Sciences on October 13th, 2005, Dr. Debasis Mitra, Vice President of the Mathematical Sciences Research Center at Bell Laboratories, Lucent Technologies, stated that real world problems motivate research. Scientists and engineers want to solve the world's challenges and must be educated for that endeavor. His advice to mathematics educators is to spend more time modeling, so we can solve problems. Chris Dede, of the Harvard Graduate School of Education, writes that "the important issue in effectiveness for learning is not the sophistication of the technologies, but the ways in which their capabilities aid and motivate users." Dede feels that researchers in higher education settings should explore the potential of emerging technologies while minimizing any unintended and negative outcomes. Isn't this what we want? Find the right way to use the laptop in the classroom to stimulate thinking among the students. In her paper entitled, "Reclaiming Real 'Basic Skills' in Mathematics Education," Nakonia Hayes discusses a publication by the National Council of Supervisors of Mathematics, which was drafted in 1977. She writes that "the changing needs of society, the explosion of the amount of quantitative data, and the availability of computers and calculators demand a refining of the priorities for basic mathematics skills." She then lists ten areas of basic skills. The first is problem solving! The tenth is computer literacy.

In July 2004, the Mathematics Standards Study Group (MSSG) met for a workshop. Many of those educators in the group were researchers who part of the Association of State Supervisors of Mathematics as well as members of the National Council of Teachers of Mathematics. They produced a document entitled, "What is Important in School Mathematics." Although the report is written for K-12 educators, the authors put forth principles needed for successful preparation of students looking to attend college. In particular, they concentrated on guidelines for early student learning to be on the right path for future learning in school and college. Mathematical knowledge should be cumulative. The MSSG urges that "the essence of mathematical learning is the process of understanding each new layer of knowledge and thoroughly mastering that knowledge in order to be able to understand the next layer." This sounds a lot like building fundamental skills. They propose that mathematical reasoning become one of the most important goals of a school education. While difficult to assess, "it must permeate all mathematical instruction." Reasoning, they argue, is a basic life skill that is as useful as arithmetic but harder to learn. Reasoning requires some amount of insight, and, as stated earlier, a CAS can offer several ways to gain insight. As an example, how many high school seniors have learned to solve three equations with three unknowns graphically? Without a CAS, they could use substitution and tediously work their way through the algebra. However, what does it mean to solve three equations with three unknowns graphically? Each equation in three variables represents a plane in space. A CAS can plot each plane, or all three planes at once (hard to do effectively by hand in a short amount of class time). Using a computer, a student can gain insight into the solution to the three equations as the intersection of the three planes. If they are linearly independent, they will intersect in a point. The graphical technique gives insight into the solution. Using a CAS, students can use various tools to analytically solve the equations (Gaussian elimination, coefficient matrix inversion, "Solve," etc.), that are far more efficient than solving by hand. A CAS can provide a great row picture or column picture of the set of equations (to use linear algebra verbiage). In a sense, the computer allows the student to see the equations in the matrix, welcoming them into the "Real World."

There has been a pronounced shift in pedagogy in the mathematics community from teaching mathematics to teaching mathematical modeling, problem solving, and critical thinking. Mathematics has become a process of transforming a problem into another form in order to another form in order to gain valuable insight about the original problem. In this way of thinking, using a CAS further allows us to build the insight. The general educational goal of the United States Military Academy is "to enable its graduates to anticipate and to respond effectively to the uncertainties of a changing technological, social, political, and economic world." Each academic department should focus on helping cadets to attain this goal. In the Department of Mathematical Sciences, we should develop the critical thinking skills of cadets to solve problems. Using a laptop computer with a CAS is an integral part in that development. No single academic department owns the teach-cadets-to-solve-problems mission; rather, all departments should work towards accomplishing this important goal. The computer can be used to further exploration, experimentation, and discovery.

Back to those skills listed in the web search. Earlier I mentioned that students in mathematics classes should learn how to understand mathematics. Given an applied problem, students should be able to develop a proper model which can produce a solution to the problem. If the solution involves evaluating a derivative or integral, the mechanics of evaluation are not as important as having the correct model. There is no need for students to memorize every integral evaluation technique (power rule, integration by parts, trigonometric substitution, u-substitution, partial fractions, numerical integration, approximation using polygons, guessing, etc.). If the student understands that the solution to the problem involves accumulation (evaluating an integral), and the student can set up the proper integral expression, why not allow the students to use a CAS for evaluation? We allow calculators to multiply large numbers. Once a solution is obtained, the critical thinking kicks in again, and the student must use reasoning to determine if the solution is valid. Graphical analysis (again produced with the assistance of the CAS) is a perfect tool for this endeavor.

As a final thought, George Polya and Gabor Szego once wrote that "an idea which can be used once is a trick. If it can be used more than once it becomes a method." Using the laptop computer in the mathematics classroom to solve problems has become a method. Allow the students to explore concepts and build their fundamental skills through insight. Empower students to solve problems with a solid foundation in reasoning and fundamental skills. Once students internalize a concept, whether by hand or with a computer algebra system, it becomes theirs forever. It is not a trick.

References

Bransford, John D., Ann L. Brown and Rodney R. Cocking, editors, *How People Learn: Brain, Mind, Experience, and School*, sponsored by the Commission on Behavioral and Social Sciences and Education. National Research Council, National Academy Press, Washington, DC, 2000.

Dede, Chris, "Vignettes About the Future of Learning Technologies,", in *2020 Visions, Transforming Education and Training Through Advanced Technologies*, September 12, 2002, a report prepared by the National Science and Technology Council found online at http://www.visions2020.gov/reports/TechPolicy/2020Visions.pdf, (accessed 28 Oct 05).

Department of Mathematical Sciences, United States Military Academy, "Required Fundamental Skills For Students Entering MA104," http://www.dean.usma.edu/departments/math/courses/ma104/Admin%20Files/REQUIRED%20FUNDAMENTAL%20SKILLS%20FOR%20STUDENTS%20ENTERING%20MA104.doc (accessed 27 Oct 2005).

Euler, Leonhard, *Introductio in Analysin Infinitorum,* Lausannae: Apud Marcum-Michaelum Bousquet & socios., 1748.

Hayes, Nakonia, "Reclaiming Real 'Basic Skills' in Mathematics Education," http://www.newhorizons.org/trans/hayes%202.htm (accessed 28 Oct 2005).

Heidenberg, Alex and Michael Huber, "Assessing the Use of Technology and Using Technology to Assess: A Case Study," Supporting Assessment in Undergraduate Mathematics, Mathematics Association of America. http://www.maa.org/saum/new_cases/new_case_08_04/USMAr3.html (accessed 28 Oct 2005).

Huber, Michael, "A Boundary-Layer Model of Thermocapillary Flow in a Cold Corner," Ph. D. dissertation, The Naval Postgraduate School, Monterey, California, 2000. Many of the quotations of mathematicians are cited here.

Mathematics Standards Study Group, "What is Important in School Mathematics," a report for Preparing Mathematicians to Educate Teachers (PMET), funded by DUE-0230847, a grant issued through the National Dissemination Track of the NSF Course, Curriculum and Laboratory Improvement Program (CCLI-ND).
http://www.maa.org/pmet/resources/MSSG_important.html (accessed 28 Oct 2005).

WordNet, on online dictionary found at http://wordnet.princeton.edu/perl/webwn (accessed 27 Oct 2005).

Lessons Learned from Running a Problem-Based Course at USAFA

Dr. Bradley Warner
Lt Col Lem Myers
Department of Mathematical Sciences, USAFA

I. Introduction

We shall discuss a method of instruction used at the United States Air Force Academy where the cadets are put in the position of interacting as junior analysts in an Air Force "analysis-shop". The purpose of this paper is to describe the objectives of the course, the course content, the course assessments, and assessment of the course.

The basic thesis of this paper is that a traditional instructor-centered classroom [1] does not enhance, promote, or develop to the full extent possible the skills needed by Air Force officers. We wanted to alter a traditional course and adapt it to reflect the expectations a young lieutenant would experience in the Air Force understanding that we would be limited by the artificialities introduced by the classroom. We felt that students working on ill-defined problems in teams where interaction with other students, the use of resources other than the textbook, and the availability of a mentor would produce students better suited for the demands of the Air Force. We hope the other service academies will evaluate these ideas and adapt and apply as they see fit.

II. Course Design and Content

The course that we selected for this project is entitled Probabilistic Models in Operations Research. This course had 115 students and three instructors. All the student were in the spring semester of their junior year and all were taking the course for their major. The three majors represented in the course were Operations Research, Systems Engineering, and Systems Engineering Management. The textbook used was Introduction to Operations Research, 8th Ed., by Hillier and Lieberman although we did not rely on it as heavily as we would in a traditional course.

We will facilitate the discussion of this course by describing its design, content and objective in the context of the instructional principles described by Saverey and Duffy [2].

1. Anchor all learning activities to a larger task or problem. Instead of proceeding through the textbook section by section we solved one ill-defined, realistic problem for each major topic. This required that we reduce the number of topics to three and that the importance of the textbook and lectures be downplayed. The three topics we selected were Discrete Markov Chains, Continuous Markov Chains (Queueing Theory), and Reliability Theory. The problem was introduced early and drove the material discussed. Thus we only used portions of the material covered in the textbook and supplemented each topic with other resources such as journal articles and other textbooks. We also required the students to spiral back to their earlier statistics and math courses. The idea was to make each topic an analysis project. For the remainder of this paper we will limit our discussion for convenience to the Queueing Theory topic.

2. Design an authentic task. For the Queueing Theory topic the students were told that the computer help desk was concerned with poor customer service related to its phone based help line and we were asked to help with the problem. Current operations were described and the students were told that some historical summary data (means and standard deviations only) such as busy rate and timing of incoming calls was provided. To increase the realism the student were told that the complete original data was no longer available. The students were told that a systems modification was put in place to increase the line capacity but that customer complaints were still an issue. A program that could simulate the call center was developed so that students could explore the problem, collect their own data, and evaluate the efficacy of their own proposed solutions. The students were also given some ill-defined customer goals and a point of contact (the instructor) and told to evaluate the current situation and make recommendations with alternatives in a written report.

3. Support the learner in developing ownership for the overall problem or task. The students were broken into groups and allowed to work in class as a team on the project. To understand the background material needed for the problem the students were asked to read the textbook and answer questions over a three lesson period. The instructor was only there to answer questions and clarify misconceptions. Then the students were to develop a plan to approach the problem. The instructor played the role of mentor and client. The students had six lessons for the project. Some of the specific issues the students had to determine were data collection protocols and sample sizes, appropriate metrics and summary statistics, proposal of alternatives, and tools to assess and compare the alternatives. In the process of the students working these issues the instructor, acting as a mentor, would provide guidance on resources and techniques for the students. For example, the students needed to find an exact confidence interval for a proportion. This idea was not suggested in the course text so the students were lead to a journal article that described a method. They then had to develop their own spreadsheet to build the confidence interval.

4. Design the task and the learning environment to reflect the complexity of the environment they should be able to function in at the end of learning. The students were asked to work in groups not of their choosing with a client that was not well versed in the topic. Their work was to result in a professional quality report to the commander of the work center. There was no written test on this material. The only assessment of their work on the topic was the written report, critiques from their teammates, a self-critique, and an oral board given by an instructor other than their own. The Air Force environment they will work in upon graduation will not have timed exams at the conclusion of a project. They will write reports, give briefings, and eventually provide input and receive feedback for a performance report.

5. Give the learner ownership of the process used to develop a solution. We tried to allow flexibility for the students to develop their own course of action. However, our attempt might have been too structured as we provided too much framework. On the Queuing Theory Project we told them to collect a data sample, gave them a paper with information on how to calculate a sample size, and then gave them explicit instructions on how to develop an Excel spreadsheet to perform the calculations. Thus we provided a procedure for them instead of having them find the path to a solution. This is the tradeoff every instructor faces between time and self-discovery. We felt it was necessary to get them going in the right direction soon or they would waste too much time and become frustrated. This did detract from them taking complete ownership of the problem.

6. Design the learning environment to support and challenge the learner's thinking. The role of the instructor was the key in this course. The instructor was to play the role of mentor/coach, client, and consultant. The students' thinking and questions drove the class. The three instructors would meet for one hour before each lesson to discuss common student questions, appropriate answers, and directions to try and steer the groups. The instructors were all experienced and highly qualified. Because of the fluid nature of this type of instruction it is hard to imagine an inexperienced instructor being able to facilitate the course.

7. Encourage testing ideas against alternative views and alternative contexts. The projects were set up so that there was no single "right" solution. In the Queueing Theory project teams could explore solutions that involved human resources, or technology implementations, as well as economic and social conditions. Of course there were some obvious courses of actions and we had to encourage further exploration by requiring all teams to provide three alternatives as solutions.

8. Provide opportunity for and support reflection on both the content learned and the learning process. An important goal in our instruction was for each student to critique their work as well as others. As an example, before introducing the first project we had the students play a simple dice game that involved discrete Markov chains and then read a professional journal article that analyzed the game. The students were asked to critique the paper with guidelines provided by the paper "How to Write About Operations Research" [3], which asked them to evaluate how well the author answered these questions:

What is the problem?
What questions do you want to address?
What part of this problem has been addressed in the past and what methods were used?
What are you doing to solve this problem?
How did you check your results?
What questions are still unanswered?

For many students this was the first time they were asked to question an official course resource and they were somewhat reluctant to do so. In the end they provided many good insights into how the paper could be improved.

III. Course Assessment

Most of the assessment instruments used in this course have already been mentioned in the previous section. However, in this section we will provide more detail about the instruments. In describing the course assessments it is important to explain the objectives. We decided to not include the more traditional objectives, such as "Find the steady state probabilities of a given transition matrix", in the course syllabus but instead to focus on high level overarching goals. For this course the objectives were:

1) Ask insightful questions.
2) Use and find different resources.
3) Understand the assumptions behind each model and know what types of problems amenable to the models.
4) Decompose a problem so that you understand what is being asked.
5) Find multiple solutions to your problem and select the best solution.
6) Present and defend in an oral board and written format your results.
7) Critique your own work as well as other authors' works.

We still had the more traditional skills level goals in the course but we de-emphasized them by articulating them during each project. We did this because we believe that students tend to focus on the skills objectives and skip the higher level more abstract objectives.

For this course we had three written projects, two oral boards, three quizzes, four computer based homework sets, a critique, and a written final exam. We will explain each of the assessment and relate them to the course objectives.

i. Computer Based Homework, Quizzes, and Written Final. There was little lecture in the course, so the material from the book was presented by answering student questions as they attempted to answer homework questions using an on-line computer based system called *WebAssign*. The purpose of this was to get them conversant with the material basics needed for the project. The students would then have a skills based quiz on the material. We were required to give a written final and so it was skills based as well, similar to the quizzes. These are traditional course assessments yet they only accounted for 35% of the course grade. These instruments allowed us to stress and assess skills objectives. Their performance on these instruments was similar to results in previous semesters on the same type of assessments. The students are well-versed in how to prepare and succeed on these types of instruments. This was in their comfort zone.

ii. Writing Assignments. These were the primary instrument used to assess the high level goals listed in the syllabus. They accounted for 33% of the overall grade. Each student had to write their own paper for each project, except the last, to include some unique contribution. They were given the grading rubric prior to the completion of the project so they would understand how they were to be graded. They were required to develop their own solutions, provide alternatives, explain assumptions, and verify results. The students were also required to grade each group member using a rubric.

In grading these assignments we used a holistic rubric, included in the appendix, which broke the paper into two parts, presentation and mathematics. Each of these was broken into two parts, style and technical for presentation and correctness and reasoning for mathematics. We noticed a marked increase in student performance over the semester. This was due in part to the fact on the last paper we let each group submit one paper. We did this because we found procrastination to be a problem, the lack of ownership described earlier, which resulted in last minute writing and subsequently poor results.

iii. Oral Boards. The most unique aspect of the course was the oral boards. Each student had to complete two oral boards, a mid-term and final. The mid-term oral board covered the first half of the course, and the final oral board was comprehensive. The students were given the questions in advance. The final oral board was administered by a different instructor from the one that the student had in class. We decided to use this assessment since we know that Air Force officers frequently give briefs where they are required to articulate and defend their ideas. Students that studied appropriately for these assessments honed their ability to think critically about the questions we were asking. They were also forced to "think on their feet" since they could not know the nuances of every question they might be asked. The instructors were able to gauge each student's fundamental understanding of the material and ask follow-up questions to press even further. These were an invaluable instrument in assessing the goals listed at the beginning of the section.

This was the first time that oral boards were run in a course with multiple instructors. The students did significantly worse on the second oral board, 71% average with 18% standard deviation, as compared to the first, 81% average with a 10% standard deviation. This may have been due to the students' lack of familiarity with the oral board facilitator. This led to questions about consistency between instructors and so the three instructors went through the results of an entire section and reviewed the scores. The conclusion was that all three were grading consistently. While the rubrics and experienced instructors made the oral boards successful, these oral boards would be extremely difficult to conduct with new faculty members. There was also the perception on the instructors' part of a large time commitment.

The Good, the Bad, and the Ugly (Assessment of the course and future plans)

This was such a radical departure from any of the students' previous courses that we spent a great deal of time trying to set realistic expectations for the course, telling the students that it would be different and how. We gave them detailed grading rubrics and assignments. We assisted them in finding other resources and acted as mentors as they developed solutions. In addition, the final grade distribution was similar to previous semesters when a more traditional course was given. Despite all these efforts the students hated the course. The end of course critique data was the worst any of the three instructors had ever received. Comments such as "The only thing I learned in this course is that I can survive anything for 42 lessons" were commonplace. The students did not perceive much value in running the course this way. Change will always lead to discomfort and some dissatisfaction; however, we believe that the students must see some value in the change. That we did not achieve this was a huge disappointment.

From an instructor's point of view, we felt that the majority of the students improved with respect to the high level objectives especially in their abilities to communicate in a written and oral format. We had some of the better students report that they appreciated this different approach as it challenged them in ways that they had never been challenged before. The instructors spent a great deal of time on this course, more than a traditional lecture course and this caused them some frustration. This course would not have been possible if it had not had experienced senior instructors thus there are questions on whether it could be sustained in its current format.

The following spring (2006) this course was offered again. We brought in some junior instructors and adapted the course to address perceived shortcomings/frustrations. We re-tooled all the assessment instruments and the structure of the course, but wanted to keep the modeling approach but needed to invest more time in developing the students' problem solving skills. There was little lecture in the course, so the material from the book was presented by answering student questions as they attempted to do the on-line homework. WebAssign was used for four-to-five lessons at the beginning of each block to introduce and generate interest in new material. This "application" approach was very successful for the majority of students in terms of familiarizing them with the subject matter and using the textbook as a reference. Unfortunately, a large number of students did not have a good set of problem-solving habits when they entered this course. They could not interpret/decipher the text, and had not learned past foundational material to the extent necessary to "get them going" on solving many of the problems. For the second offering of the course, we set aside one or two lessons at the beginning of each block (before starting WebAssign) to discuss strategies of effective problem-solving within the constraints of this course material. This included dynamic whole-class discussions of the foundational topics in the text, effective use of the setup of the text (i.e. index, table of contents, etc.), and sound problem-solving strategies (i.e. Polya's problem-solving checklist [4]: 1. understand the problem, 2. devise a plan, 3. carry out the plan, and 4. look back at solution). The students seemed much more comfortable with the presentation of the material even though the change was actually quite small.

For the oral boards we noticed that many students were ill-equipped to prepare for these assessments. Not only had most students never experienced this type of assessment, but the bulk of the students' time in class was spent on problem-solving (WebAssign homework and projects) versus deep discussions about the mathematical material which was necessary to prepare for these boards. The expectation for what constituted an "acceptable" presentation was inadequately fleshed out (if this is even possible to flesh out), as was perhaps evidenced by the large variance between instructor assessment scores. During the follow-on semester we dedicated a large portion of class time to preparing for oral examinations (for example, presenting a mock oral examination where the teacher played the role of student). In the same way it was suggested earlier to prepare for using the WebAssign effectively, we attempted to teach them the skills required to succeed at this type of event (critical discussion of material, analyzing their own understanding, etc.). The second oral board was changed significantly for the second offering of the course in that we had "group panels." The group evaluations were identical to the individual oral boards of the previous semester, but focused exclusively on the project that group had been working on, and allowed us to spend more time on weaker students than on stronger students. This change was very valuable and gave us greater flexibility in the assessment process.

One quiz was originally given each block. Each quiz was intended to augment WebAssign and assess types of problems that could not be easily executed in that venue. When used to augment the problem set in WebAssign, the quizzes were a valuable way to assess skills and ideas that could not be assessed with WebAssign. The quizzes were also the only real-time written assessments that prepared them for the final exam. For the follow-on semester, we gave short exams (instead of quizzes) which very closely resembled the types of problems done in the WebAssignments. These timed events focused exclusively on problem-solving, modeling, and "fundamental computational" skills as our assessment of individual conceptual understanding came from the oral examinations. The final exam mirrored the quizzes and focused on writing ability (essay questions in line with their writing assignments) and problem-solving skills (work-out problems in line with the homework and timed assessments). The spirit of this final exam was used again for the follow-on semester.

Conclusion

The type of modeling based course that we have described in this paper has many advantages but is not without its drawbacks. The majority of the class periods had the students engaged in discussions, asking questions, and determining courses of action. There was no time for them to become passive. The typical classroom that has some students sleeping and other doodling did not happen. The projects were interesting and the reports had many thoughtful insights. On the downside the work load for the instructors was much higher and the demands on the students were so different that they did not react well to them. The course as presently structured would need earlier courses to help develop the students. Students must experience this type of classroom and these types of assessment instruments before their sixth semester of college to make a course like this successful. An analogy our Director of Assessment likes to use is that you can't place a life-long carb eater on a low-carb diet and not expect them to complain about not having any pizza. It is clear that getting more courses to adapt this modeling based teaching philosophy is difficult and will take time. Thus we have modified the course so as to maintain the modeling approach but make it less foreign to what a student expects from the class.

References

[1] Barr, Robert and Tagg, John. *From Teaching to Learning: A New Paradigm for Undergraduate Education.* http://critical.tamucc.edu/~blalock/readings/tch2learn.htm, accessed 15 November 2005.

[2] Savery, John and Duffy, Thomas. *Problem Based Learning: An instructional model and its constructivist framework.* Bloomington, Indiana: Center for Research on Learning and Technology, Indiana University, 2001.

[3] Brown, Gerald. *How to Write About Operations Research.* Phalanx, September 2004.

[4] Polya, George. *How To Solve It: A New Aspect of Mathematical Method.* Princeton University Press, Princeton, New Jersey, 1945.

Space War and Neutron Stars: Exploring Advanced Topics with Cadets at USMA

Dr. Chris Moseley
Department of Mathematical Sciences, USMA

Discussions of technology in mathematics classrooms sometimes take on an either/or quality: either require students to make full use of computer software in the classroom, including exams, or demand complete reliance on traditional, manual calculation. This paper attempts to answer the question: can students learn from a problem that makes good use of both? In the spring semester of 2005, my MA489 (Special Topics in Mathematics) cadets tackled one such problem, a problem that was "far out"—deep inside the gravitational field of a neutron star light years from Earth. Their solution confirmed the intuition of a renowned science-fiction author who had written a fictional encounter with a neutron star decades earlier.

The story of this class begins in the spring of 2004, when I remarked to my Vector Calculus students that they had learned enough mathematics to understand James Hartle's recent textbook, *Gravity: An Introduction to Einstein's General Relativity* [1]. I added that general relativity would be a good subject for a Special Topics course, hoping that perhaps one or two cadets would be interested. To my surprise, six cadets emailed me repeatedly during the fall of 2004, asking if this course would be offered! In January 2005, the cadets and I found a classroom and began our work using two texts: *Gravity* and another, somewhat unusual "textbook" that will be described presently.

General relativity is notoriously difficult, partly because traditional treatments begin with about two months of differential geometry (affine connections, the Riemann curvature tensor and other machinery) before students see any of the ``marquee" topics they've heard about, such as black holes and the expanding universe. The *Gravity* textbook takes the opposite approach: the marquee topics are introduced early, with just the right amount of differential geometry needed at each step to grasp the central ideas and perform interesting calculations. This allows one to pack a good deal of geometry into the syllabus in a fairly painless way. By the end of the term, my students knew how to derive the geodesic equations for a given metric tensor using the Euler-Lagrange equations, how to find conservation laws using Killing vector fields, and were performing orbital mechanics calculations in Schwarzschild spacetime. Many of the topics we covered required "hands on" knowledge of single-variable and multivariable calculus: for example, just to follow the derivation of the Euler-Lagrange equations for geodesics requires students to understand integration by parts.

The other assigned text for the course was not a traditional textbook at all. Years ago I had read Larry Niven's science-fiction novel *Protector* [2] and was impressed by his vivid and accurate descriptions of the effects of relativity on space travelers. I decided to assign *Protector* as required reading, to help the students gain some intuition into relativistic phenomena, and also for its sheer entertainment value.

For homework, I assigned many of the (excellent) problems in *Gravity*. However, when the time came to assign a group project for the course, it was the science-fiction novel that provided the best problem. The main event in *Protector* is a "space dogfight" in which two astronauts fly dangerously close to a slowly rotating neutron star in order to perturb their course and throw off enemy pursuit. For those who may not be familiar with the concept of a neutron star, this is an object with roughly the mass of our Sun, but so dense that it is only a few miles in diameter! The gravitational field near such a star is so intense that Newtonian physics breaks down and calculations of orbits must take the mathematics of general relativity into account.

With this in mind, I asked the cadets to compute the spacecraft's initial acceleration, braking deceleration, and the orbital elements (energy per unit mass and angular velocity) required to execute the gravitational perturbation maneuver while remaining consistent with Mr. Niven's story. The acceleration/deceleration aspect of the problem used formulas that they had earlier derived by hand as part of their homework from the *Gravity* textbook. However, to find the orbital elements needed for a successful maneuver near the neutron star, the cadets used a *Mathematica* notebook for numerically integrating orbits in Schwarzschild spacetime provided by the *Gravity* website [3]. This notebook allowed them to input different orbital elements and observe the resulting orbits plotted in a star-centered graph. In effect, they were running simulations of the encounter with the star, just as NASA would in planning a space mission. The ability to change inputs and see the results instantly was indispensable in devising an orbit that satisfied both the laws of general relativity and the plot of *Protector* (and kept the astronauts alive). The gravitational perturbation orbit they eventually designed is shown in Figure 1.

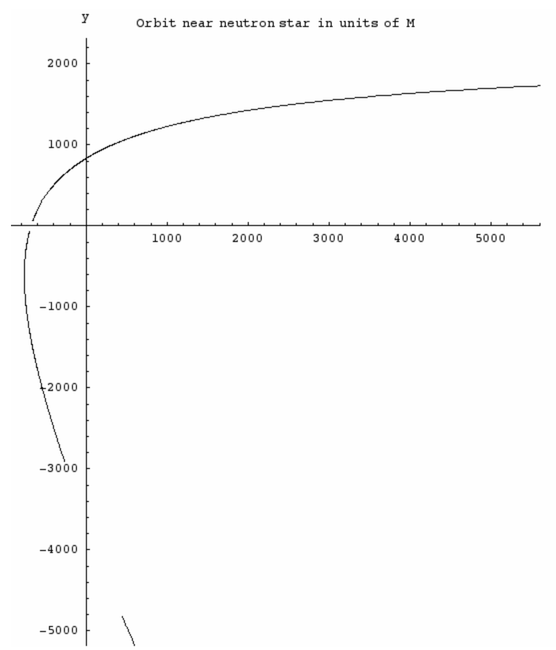

Figure 1. Escape orbit near fictional neutron star BVS-1, computed by the MA 489 cadets.

The cadets found that almost everything in the novel could be accomplished (in theory), with only a slight modification to the timeline of events. This validated Mr. Niven's remarkable intuition—he wrote the story in the 1970s, without a computer to numerically integrate orbits.

There were two aspects to the cadets' final report that bear mentioning here:

1) Many of their calculations were based on equations the cadets had derived by hand in previous homework assignments; this laid the foundation for their successful analysis of the *Protector* dogfight.

2) The use of the *Mathematica* notebook was equally important; the ability to simulate candidate flight paths near the neutron star was a crucial aid in designing an orbit that satisfied the demands of physics and the plot of the novel.

In summary: in this course we found a problem that used what students learned "by hand" and employed this knowledge to make intelligent use of a *Mathematica* notebook in their analysis. To be sure, the mathematical skills needed often went far beyond "fundamental skills," but the fundamentals were no less necessary. On the other hand, without *Mathematica* the cadets would not have been able to find a solution to the problem.

[1] Hartle, James B. *Gravity: An Introduction to Einstein's General Relativity.* San Francisco: Addison Wesley, 2003.

[2] Niven, Larry. *Protector.* New York: Ballantine Books, 1973.

[3] http://wps.aw.com/aw_hartle_gravity_1/0,6533,512494-,00.html, last accessed June 2006.