COOPERATION IN NETWORKS: MATHEMATICS OF METRICS AND UTILITY[1]

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Introduction: The Army's social networks often involve cooperation – entities working together to achieve a common goal. What are the metrics for such a network? What is cooperation and what makes cooperative networks effective? Can network structures and processes enhance cooperation and optimize networks? This presentation uses the emerging mathematical framework of *subset team games* (Arney & Peterson 2008), which provides a theory of the principles, relationships, and metrics of cooperative phenomena. The subset team game framework is focused on agents (network entities) working together for a common good, reflecting the team-oriented cooperation that characterizes social and military networks. The underlying assumption of subset team games is that players are motivated by a *combination* of selfish and altruistic reasons, and the framework provides computable metrics from the network's utility. The framework provides derived metrics of an agent's *selfish contribution*, *altruistic contribution*, and *total contribution* from a carefully defined *utility function*.

In this presentation, we explain using subset team games to understand teamwork. First, *cooperation space* provides a visual means of assessing and comparing the cooperative nature of multiple algorithms geared toward the same tasks. Second, the *cooperation complex* provides a snapshot of the contributions of all subsets of a team. Together, these tools provide a means of visualizing and comparing multiple algorithms. We illustrate their use in network flow that requires distributing information load across a network graph. The strength of the theory of subset team games is its ability to provide insight into the cooperative nature of various algorithms that accomplish the same task. Algorithms can be classified on a spectrum between "altruistic" and "selfish". While an algorithm's rate of success is important, a singular focus on this number can lead to disappointing results. This is particularly true in cooperative systems, where other ideas such as cohesion and trust play key roles. Given the choice between multiple

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[1] This work is a consolidation of previous work reported at conferences and will be published and presented in other forms this Fall and Winter. Specifically, results appear in:
Arney, D. C. & Peterson, E. (2008), Cooperation in social networks: communication, trust, and selflessness. *Proceedings of the 2008 Army Science Conference.* Orlando, FL.
Arney, D. C. & Peterson, E. (2009), Cooperation in a Network: Altruism Increases Flow, *USMA Network Science Workshop Report*, USMA Network Science Workshop.
Arney, D. C. & Peterson, E. (2010), Cooperation in a Network: Agent Metrics & Cooperation Space, *Proceedings of the 2010 Army Science Conference.* Orlando, FL.
Arney, D. C., Arney, K. and Peterson, E. (2010), Autonomous Geometric Cooperation: Pursuit-Evasion, Equi-Distribution, Surveillance, *Proceedings of the 2010 Army Science Conference*, Orlando, FL.

algorithms with equal success rates, it is better by far to choose a more altruistic algorithm since it has a more positive impact on the team. The Army's culture is based on cooperative networks. Many emerging areas of net-centric warfare involve cooperation, requiring trust and autonomy of the agents. Therefore, understanding, designing for, and using cooperation are critical elements in meeting the goals of the future Army. This fundamental research in the basic mathematical principles of cooperation can contribute greatly to that effort and these important Army goals.

Background: Traditional approaches to social network analysis are often performed either by studying individual nodes or by studying the graph as a whole (Wasserman & Faust, 1994). Beside statistical techniques, there are few theories that attempt a unified approach, encompassing both the individual and the team view. This paper implements this kind of theory using subset team games. The framework of subset team games was developed to understand cooperation *simultaneously* at the individual and the team level. In earlier work (Arney & Peterson, 2009) (Arney & Peterson, 2010), it is evident that the framework is a useful tool for understanding cooperation in a system. This is because it provides notions of both *altruistic* and *selfish cooperation*. This is important because entities in real life are not rational in the strict sense of classical cooperative game theory (Axelrod, 1984) (von Neumann, 1928) (Sally, 1995). Instead, as in the Army, teams of individuals are designed to work together for the common good. Other notions of cooperative systems are also emerging as vital to the modern Army, such as robotic teams, teams of software agents, groups of sensors, and there are additional questions regarding systems with both human and artificial agents. In this paper, we provide several examples illustrating how *subset team games* may be used as an analytical tool.

Metrics, Utility Functions, Cooperation Space: A metric (or distance function) is a function with special properties which defines distance between elements. Metrics then become utility functions for a team when they measure and evaluate the process or function of an entity or cooperative group of entities (organization or team). This utility function is then used in the determination of the condition or level of productive or mission-accomplishing activity of the organization or team. When subset utility functions are appropriately determined for the network (team), altruistic and competitive contributions provide an analytical tool for comparing the value of various subsets of a "team" of players. We call this the *cooperation map*, and say that it takes values in *cooperation space*. Figure 1 is a visual depiction of this space, along with regions marked by sensibility and cohesiveness. If all points take values in the region with $a_S \geq 0$, the utility function is said to be *cohesive* because a negative altruistic contribution indicates some players would be better off by themselves. If all points take values in the region with $c_S \geq 0$, the function is said to be *sensible*, because utility functions that do not increase with the size of the subset have limited value.

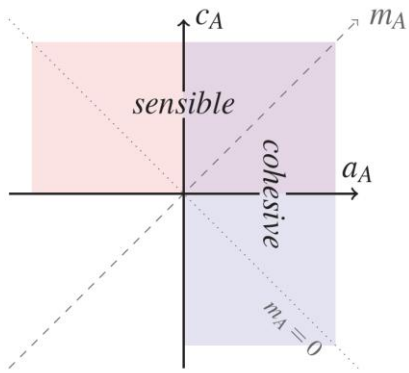


Figure 1. Cooperation space of a coalition A. Ideal behaviors take values in Quadrant I.

Example 1: Network Flow Game: In broadest terms, the cooperation space visualization depends upon an underlying algorithm or behavior, and the choice of a subset utility function (or metric). It is therefore very useful as a comparative tool for analyzing or classifying algorithms and behaviors. Figure 2 shows a simple network flow game where individual nodes share their channel capacities to pass information through a system. The objective of the system is to maximize the amount of information that reaches a destination. In this game, the altruistic contribution of nodes is well-correlated with a generic understanding of a "cooperative" algorithm (see Peterson & Arney 2008). On the other hand, "greedy" algorithms tended to have a higher selfish contribution. Building upon this work, we investigated various algorithms to determine their effectiveness, viewed through the lens of cooperation space. Altruism compares the system's effectiveness when a player is removed from the system. In this reduced system, we removed the player and its channel, and redistributed that player's load across all other available nodes. The results of a simulation with 20 nodes and 20 channels are shown in Figure 3, which compares three different algorithms. There is a substantial difference between the algorithms as viewed in cooperation space. In particular, the "proportional" algorithm in (c) differs significantly from the "even" algorithm in (a) and the "greedy" algorithm in (b).

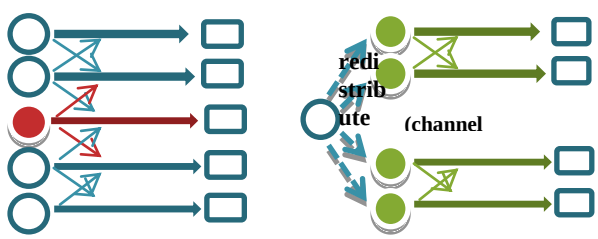
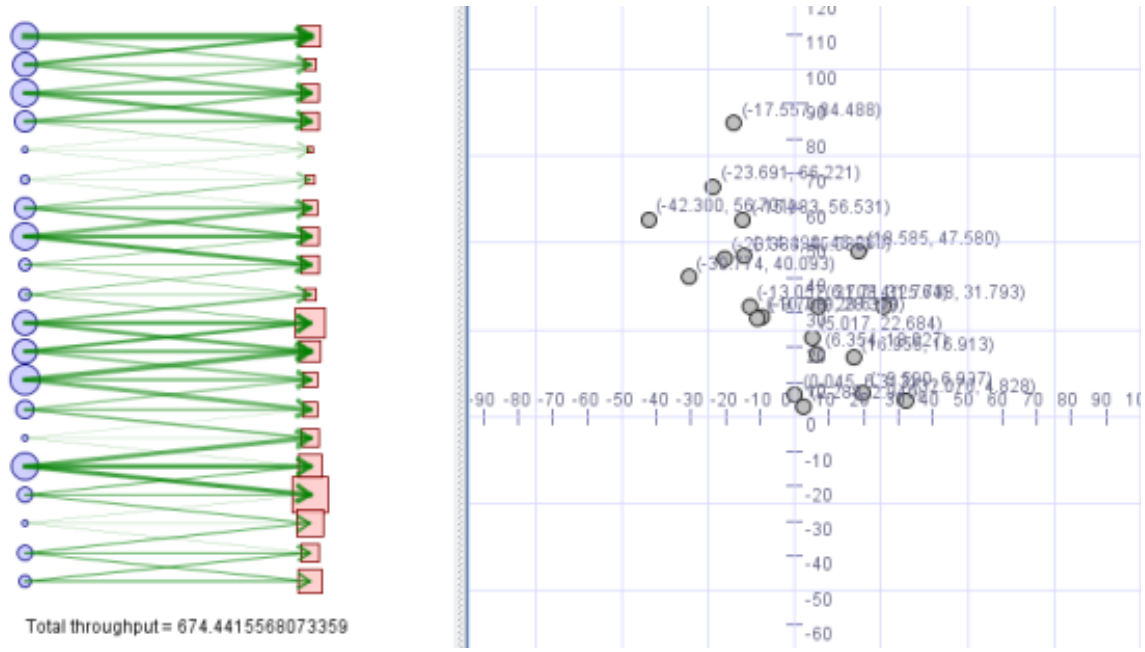


Figure 2. Network flow game. The circles represent nodes, and the rectangles represent capacities. Each node may pass information through its neighboring channels.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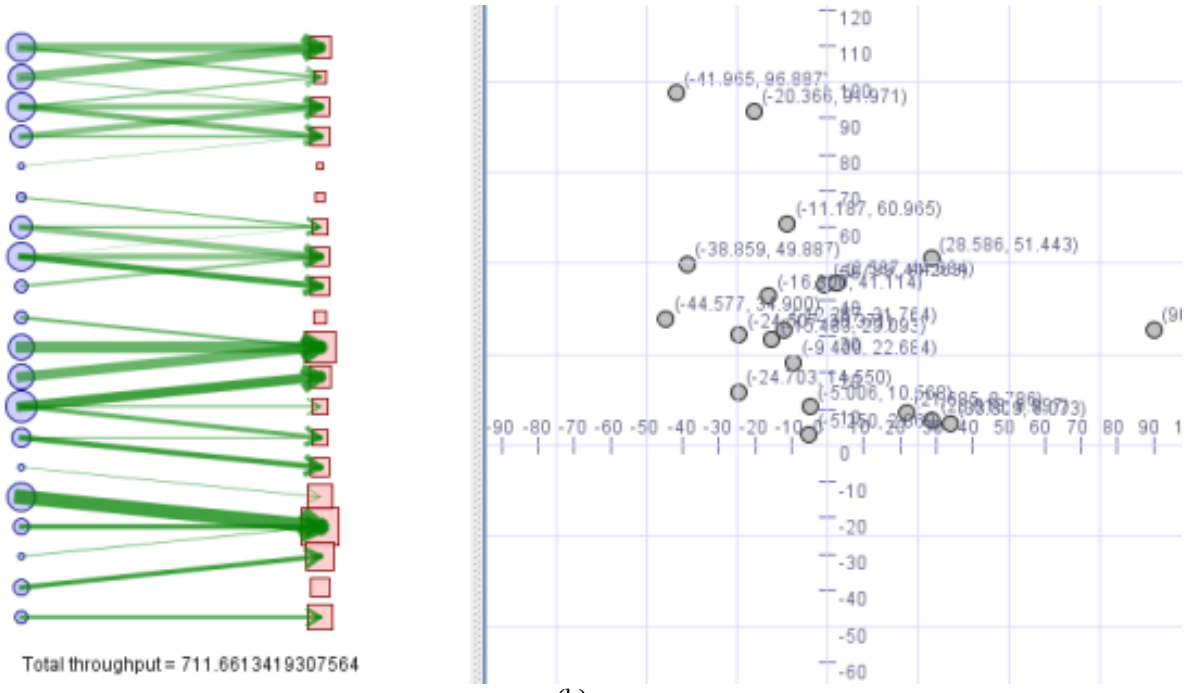
(a)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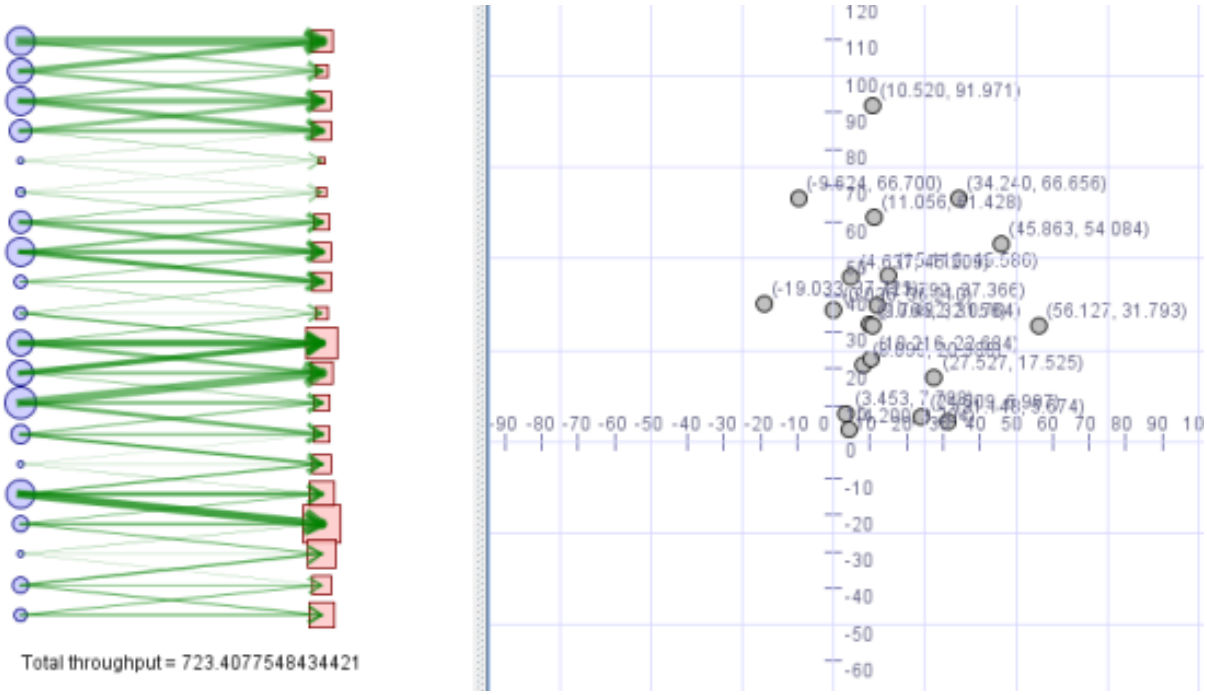
(b)



(c)

Figure 3. The results of a network flow game. In (a) nodes divide their load evenly. In (b), nodes place their loads on the highest channels. In (c), nodes assign loads to channels in proportion to their capacity. The plot shows the results in cooperation space.

Example 2: Geometric Equi-distribution problem: We use cooperation in the task of determining geometric assignments in a polygonal region to establish regions of equal area. Each entity is assigned a region within the polygon, and the goal is for the entities to sense their situation and move autonomously to equalize their assigned areas. Two agent-based algorithms are run in simulations to test performance and convergence. Metrics are computed to determine how successful the entity distribution is at any stage of the dynamics. The metrics that are used are normalized with respect to the mean area of the sub-regions and the number of entities to capture the deviations in the areas of responsibilities as ratios. Let $\boldsymbol{\bar{A}}$ represent the mean sub-region area, and let $\boldsymbol{A_1}, \boldsymbol{A_2}, \ldots, \boldsymbol{A_n}$ represent the areas assigned to players. We track 3 measures:

1) Maximum deviation from the mean as a ratio with mean sub-region area: $\max_i |A_i - \bar{A}| / \bar{A}$.
2) The average deviation from the mean as a ratio with mean sub-region area: $\frac{1}{n}\sum |A_i - \bar{A}| / \bar{A}$.
3) The sum of the mean-squared deviations as a ratio with the square of the mean sub-region area: $\frac{1}{n}\sum (A_i - \bar{A})^2 / \bar{A}^2$.

By tracking these normalized utility metrics (ratios or percentages), we are able to normalize the algorithms behavior independent of the size of the region or the number of points used in the simulation. (Arney, Arney, & Peterson 2010) The simulations are performed by establishing a polygonal region and initially randomly dispersing a specified number of points (representing players) in the region. A Voronoi diagram then partitions the overall region into sub-regions where each point is responsible for the area where it is the closest point. This geometric framework is shown as the "meshed" region in Figure 4. In a dynamic assignment game, the

areas would then be assigned to each given point as it cooperates to maximize the utility function. The simulations implement various algorithms to dynamically adjust the point locations to make the areas equal.

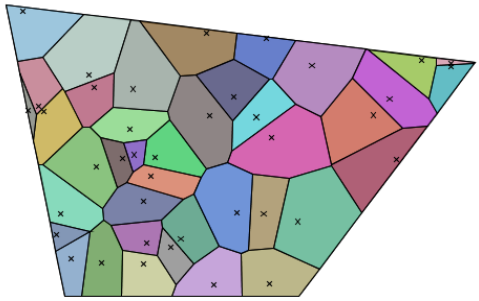


Figure 4: Initial "meshed" region with 40 points (unequal areas). This region has poor utility metrics --- e.g., the maximum assigned area for the largest entity is 140% of the mean area.

We begin with the premise that when the autonomous entities in a region sense they are no longer in balance in terms of equal areas of responsibility, they attempt to move to equalize the areas using only local information. To do this, the entity determines local factors (distances or areas or loads of neighboring entities) and seeks to move in a way to reduce the imbalance. We run two algorithms (S-1, S-2) to test their performance in various scenarios. In S-2, each entity moves in a direction found by weighting the differences in the region's area with all its adjacent neighbor areas. In S-1, each entity moves with the average of all neighbor movements (iterative scheme) if its area is close to the mean sub-region area. We did this for 40, 100, and 400 entities. An image of the region with a sample converged mesh (40 areas or sub-regions that are equal) is shown in Figure 5. The performance and convergence data for the simulations for algorithm S-2 are provided in Table 1.

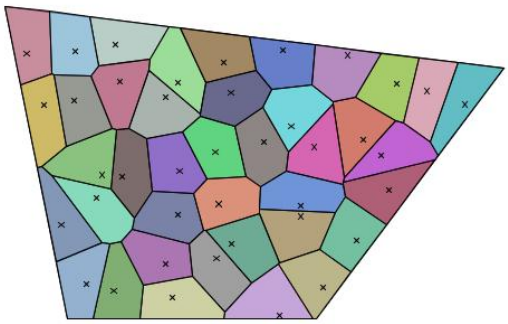


Figure 5: Converged region with 40 points arrayed in locations to produce equal areas of responsibility.

Table 1: Performance metrics for algorithm S-2.

			Number of Steps				
		Initial	**10**	**50**	**100**	**250**	**500**
CASE 1: 40 points	**Maximum Deviation**	1.417	0.332	0.057	0.018	0.002	1.20E-04
	Mean Deviation	0.429	0.094	0.011	0.004	2.97E-04	2.18E-05
	Mean Sq. Deviation	0.292	0.014	2.47E-04	2.62E-05	2.48E-07	1.25E-09
CASE 2: 40 points	**Maximum Deviation**	1.191	0.167	0.114	0.069	1.66E-04	2.18E-07
	Mean Deviation	0.303	0.051	0.012	0.009	4.65E-05	6.47E-08
	Mean Sq. Deviation	0.147	0.004	5.96E-04	2.88E-04	3.70E-09	7.58E-15
100 points	**Maximum Deviation**	2.35	0.521	0.288	0.064	0.037	0.052
	Mean Deviation	0.474	0.152	0.063	0.024	0.005	0.004
	Mean Sq. Deviation	0.362	0.037	0.006	7.93E-04	5.42E-05	7.18E-05
400 points	**Maximum Deviation**	2.572	1.028	0.437	0.389	0.257	0.190
	Mean Deviation	0.428	0.218	0.093	0.054	0.014	0.014
	Mean Sq. Deviation	0.314	0.078	0.013	0.005	0.001	5.04E-04

In the following simulation, different individual entities were programmed to follow different algorithms forming teams whose goal was to converge (the utility function measures how close each entity is to the mean or equi-distribution). The entities were scored as to their altruistic and selfish contributions and teams were scored as to their overall performance. Figure 6 shows the evolution of average team scores for five teams of various player (algorithm) composition, with a maximum/ideal score of 20 and the average taken over 200 sets of initial positions.

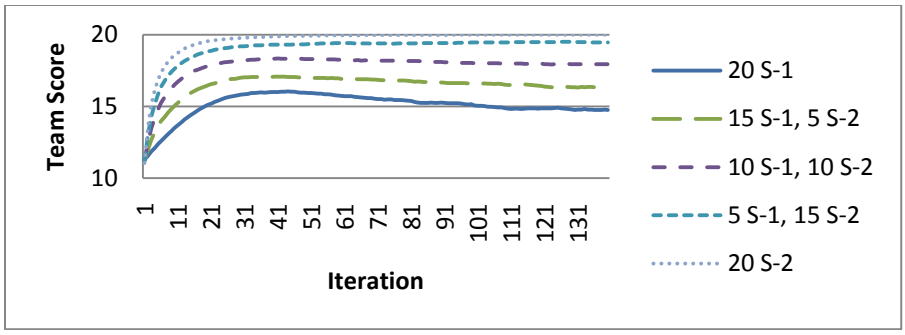


Figure 6: Team scores reflecting average closeness to equi-distribution for five teams of 20 players with various compositions of players using the S-1 and S-2 algorithms (as indicated). Each line represents the average of 200 runs for different sets of initial conditions, with each run consisting of 140 iterations.

It is clear from Figure 6 that the S-2 algorithm performs better on a homogeneous team than the S-1 algorithm, but it is less clear to what extent the individual players (algorithms) contribute to the overall team score on a heterogeneous team. On this individual level, the results indicate that the S-2 algorithms perform better than the S-1 algorithms on heterogeneous teams, in *both* their selfish and altruistic contribution. Figure 7 shows a comparison of the altruistic and selfish scores of the S-1 and S-2 players on a common team, in this case a team of 20 entities with 10 S-1 and 10 S-2 players. The selfish contribution of S-2 is better on average, and the gap widens over time. The altruistic contribution of S-2 is also better on average, and increases over time. Similar differences in cooperation utility were also apparent in all other team compositions of players performing the S-1 and S-2 algorithms. These results indicate that the S-2 algorithm, which weights individual movements based on differences in neighbor areas, improves the overall team score beyond what is expected from a single individual gain. Even in the scenario where almost all players follow S-1, a few S-2 players can have a positive impact on their fellow S-1 players.

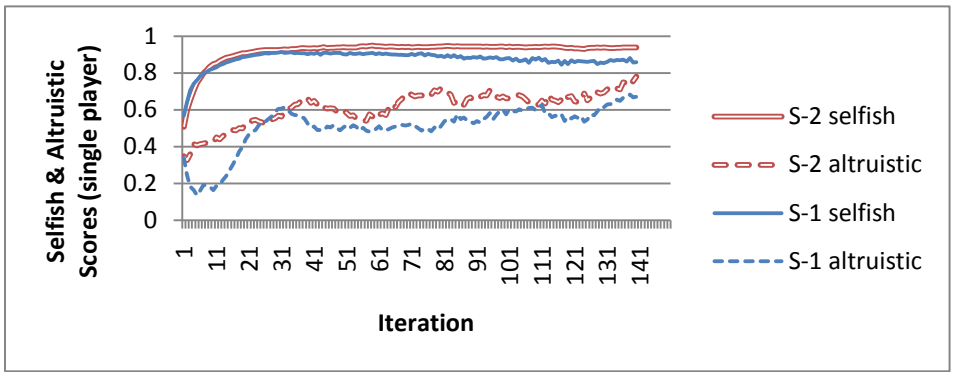


Figure 7: Selfish and altruistic scores for players operating with S-1 and S-2 algorithms on a team with 10 S-1 players and 10 S-2 players. Each line represents the average of 200 runs for different initial conditions, with each run consisting of 140 iterations. Despite noise in the results, the S-2 algorithm is more altruistic.

Future Work: We have described the concept of subset team games, and illustrated analysis of several simple games using the utility functions and metrics of cooperation provided by that framework. We assumed that the subset utility function was given exogenously, making the framework a primarily explanatory model for the nature and value of cooperation of subsets of players. Other games that have been studied using these methods include hypothetical basketball games (Arney and Peterson 2008) and pursuit and evasion games (see Gebhart 2009).

These results are intended to increase the understanding of cooperation from an analytical perspective. Much work needs to be done to determine to what extent these ideas will translate to applications. The strongest point of impact is probably the understanding of metrics of trust. With trust defined as the confidence that one player has that another player will behave in a way that benefits the team, it makes sense to use the altruistic cooperation score to build trust and ultimately measure trust. The score that most closely measures this within our framework is altruism. A player which has a high marginal contribution but is primarily selfish may not be working for the team and therefore cannot be trusted as much as an altruistic player.

A secondary point of application might be network disruption. Because the metric of altruism is also connected closely with the idea of cohesion, it follows that the optimal points of disruption are precisely the points with the highest altruistic score. Again, this idea depends fundamentally on the definition of a subset utility, and needs to be tested further.

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