Social Capital in Dynamic Social Networks

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1. Introduction

Dynamic nature of Social networks is attributed to volatility of underpinning attributes that promote explicit as well as implicit links between pairs of networked individuals. More than social and physical resources surrounding networked groups, Social capital is the purpose for formation and maintenance of social networks. Despite Robert Putnam's conceptualizing for social capital (1996) and (2000), it remains elusive. We posit that we can begin to understand social capital in Social Psychology constituents of it including interpersonal trust, autonomy, power, and norms.

A determinant of social capital is interpersonal trust. Through a series of publications (Hexmoor, et. al., 2006) and (Hexmoor, 2009), we have marshaled numerous novel conceptions of computational trust from interpersonal to institutional. Another dominant determinant for social capital is autonomy. We have organized a series of six annual workshops at the premiere agents conference AAMAS that led to a journal special issue (Hexmoor, 2003) and a heavily cited edited book (Hexmoor, et. al., 2003). Other determinants we have explored are social norms (Hexmoor and Hayes, 2006) and power (McLaughlan and Hexmoor, 2006, and Hayes and Hexmoor, 2006), yet much remains to be explored.

We believe social networks of interest to the military share common social capital that can characterize them for identification and tracking as they evolve. For example, in contrast to traditional military hierarchy, cellular organizations have low power index; there is high level of autonomy and trust within cells. Social norms and policies are responsible for maintaining social order (López-Pérez, 2007). *Web of trust* is a network centric concept used in cyber security, user authenticity to be specific (Ferguson, and Schneier, 2003) and service level agreements (SLA) codify organizational conventions and commitments. For example, SLA may specify the levels of availability, serviceability, performance, operation, or other attributes of the service and penalties (Keller, et. al., 2003). In summary, social networks are vessels for the maintaining social fabric of society and, social networks contain the following:

a. Social norms and policies of their respective constituents
b. Social networks maintain webs of trust and power
c. Organizational cohesion for cooperation and coordination (SLA)

2. A Formal Model

Let N = {1, 2, …, n} be a finite population of individuals who interact repeatedly in pairs by their participation in a social network. Each agent will be randomly assigned *k* unary attributes (e.g., actor type or ideology) that will affect their affinity, i.e., mutual attraction, to others. Attributes are used to determine *homophily* among individuals (Jackson, 2008). We will use *Jaccard similarity coefficient* (Lipkus, 1999) to compute the amount of homophily between pairs

of agents. For simplicity, we will assume attributes to stay constant over time. In reality, attributes change over time and these changes account for changes in homophily among individuals and in turn they produce links that change over time. As individuals change naturally, their gravitational force for attraction changes accordingly; these dynamic considerations are outside our current scope. While homophily might run counter to some social psychology evidence for attraction among opposites (Lieberman, et. al., 2007), we will follow the general trend to assume this holds until we consider more complex settings. The canonical social network for N will be an undirected graph $g \subset N * N$. In this formulation two nodes are either connected or they are not. It cannot be that one node is related to a second without the second being related to the first.

Initially, we randomly assign each actor to one of two political stances (i.e., opinions) – (a) for violence (i.e., hawkish using Economics terms), or (b) against violence (i.e., pacifist or "dove" using Economics terms). These differ from individual attributes and will be used as messages to be sent to other individuals. In a given time cycle, a *message* is transmitted through a link between nodes A and B only once. Abandoning directional links helps us to avoid complexity of cycles. Although a link lacks direction, in a given time unit, a message is sent once from a sender to a receiver without concern for acknowledgement. For simplicity, we do not consider transmission failures of any type.

Given the set N of agents, connection between each pair of agents is initially guided by a parametric, stochastic value. Links will be bidirectional [If discussing a different model then a better transition must be made between models or tactics but directionless links are discussed first followed by bidirectional links] but lack reflexivity. *Links* are used as the exclusive channels for information exchange. Consider each bilateral interaction as the development of a shared project over time. Given exogenous conditions, interactions will exhibit both long-run benefits from cooperation and short-run gains from opportunistic behavior.

Frequency of usage is the first property—we assume that the more often a link is used, the stronger it becomes. The second property is homophily. Pairs of connected agents who share multiple attributes will strengthen their link proportional to the force of their homophily. Pairs of connected agents who share multiple attributes will strengthen their link by force of homophily, while other links might be weakened or dissolved due to lack of adequate homophily, thus allowing us to develop networks that change shape over time. A link might contribute to *bonding*, *bridging*, *homogeneity*, or *heterogeneity* of its nodes. Each of these will be used as dynamically varying attributes. Link AB has increased *bondiness* iff A or B exchange previously held opinions. Increased *bridginess* comes from exchanges of fresher opinions.

Transitivity is an individual node property that will be set to on or off. With it on, messages move forward one unit automatically beyond the recipient but proportional to link strength. This is represented by a coefficient 1/L. If a link to a node is an indirect link, strength of that link is multiplied by 1/L where L is a user determined attenuation parameter. Homogeneity is increased when transitive opinions are exchanged. In contrast, increased heterogeneity is due to exchange of opinions that have not been transitively transported.

Transitivity is a global link property that can be set for every node as a toggle switch value; i.e., all links on or all links off. With transitivity set to off, information exchange is dyadic and no

indirect links exist. Influence is limited to direct communication. Information received stops going forward beyond the message receiver.

With it set to on, transitive information exchange is hyperdyadic, i.e., going beyond a pair of communicating nodes. For simplicity, we limit transitivity to a single link. Therefore, influence in one time unit is limited to "friend of a friend" and not beyond.

Network architecture, i.e., *topology* or *structure* is the user-determined shape of the network. . The two topologies we will initially explore are random and scale free networks (Newman, et. al., 2006). Scale free networks were first proposed by Albert Barabasi to reflect the *power law* distribution of connectivity (Barabasi, et. al., 1999). Beyond initial individual stances, we model changes to these individual states as a function of network interactions that are modeled in terms of simulated exchanges of political stance among linked pairs.

Bonacich centrality measure of an agent is the degree of the agent's embeddedness in the network that accounts for [the connections of those connected to the agent] (Bonachich, 1987). This is a weighted count of the number of walks originating (or terminating) at a given node. The walks are weighted inversely by their length so that, long highly indirect walks count for little, while short, direct walks count for a great deal.

(1) $w_{ij} = \delta(a_{ij} + b(a^2)_{ij} + b^2(a^3)_{ij} + \cdot\cdot\cdot) = \delta \sum_{k=0}^{\infty} b^k (a^{k+1})_{ij}$

In equation 1, the number of walks of length *k* between all pairs of nodes is given by the *k*th power of the adjacency matrix. b is chosen to be smaller than the reciprocal of the largest eigenvalue of adjacency matrix A. Each agent i node in the network will have a known Bonacich value at time t, denoted B_i^t. The strength of a link between a pair of agents i and j is denoted by S_{ij}. Peer influence is gathered by the agent via connections. Influences are given by sum of opinions of an agent's n neighbors weighted by the link strength to neighbors and their Bonacich values, shown in equation 2.

(2) $\text{Influence}^{t+1} = \sum_{ij=0}^{n} B_i^t \, S_{ij} \text{Opinion}^t$

Opinion influence in 2 reflects the amount of influence on an individual from others. B_i^t is the Bonacich value and S_{ij} is the strength of connection between i and j. The *behavioral intention* of an agent is shown in equation 3 to be dependent on a combination of endogenous *attitude* (i.e.,the first term in 3), Influence (i.e., the second term in 3), and sum of m norms (i.e., third term in 3) that affect an agent's intentions. Attitudes of entire population can be set to a scale, say 1-7. Every agent is assigned an independent error for her *intention*. We assign the error to agents in the range of -1 to +1 resembling a bell curve with most of them having an error of zero. This error term accounts for the inaccuracies across individuals. The style of behavioral intention equation is similar to theory of reasoned action from social psychology (Ajzen, and Fishbein, 1975).

(3) Behavioral intention =Weight on Attitude * Attitude + Weight on Influence * OpinionInfluence + $\sum_{k=1}^{m}$ Weight on Subjective Norm_i * Subjective Norm_i + Error $_{i.}$

This model will be studied using a period of parametric time period, T, that will be used to simulate T distinct exchanges. It is commonly known that influence among individuals dissipates after three degrees of separation (Christakis and Fowler, 2009), therefore T will be assigned a number > 3. Within $(n-1)*T$ time units, information will reach the periphery of the network. With hypothetically X nodes (X < n) leading original opinions, we will need X* (n-1) * T time units to inform all nodes. If X = n, initial time spread phase will be n* (n-1) * T. Transitivity will accelerate dissemination.

In dyadic communication, Contagion will be limited to neighbors (i.e., one degree of separation), thus ensuring that the network is part of the evolution. Individuals will increment opinions they receive from their neighbors. If the opinions are the same as their own, they strengthen their original stance. Otherwise, if the balance of opinions overwhelmingly differs from their own, they will change. A global, default parameter will be used to determine the threshold of opinion stubbornness in the population.

At each node and each cycle of simulation, we gather influence of others by adding up direct and indirect influences of others. For all direct links of a node we add hawk and dove messages. Balance of this value reflects the sum of direct influences. For all indirect links of a node we add hawk and dove messages. Balance of this value multiplied by 1/L reflects the sum of indirect influences. In reality, there will be a natural temporal decay in link strengths as well as influences at each node. However, for simplicity, we will assume that there is no decay. Keeping all else equal, we plan to test the salient hypotheses with our agent-based simulations.

Conclusions

At the Nash equilibrium, Individual opinion outcomes as well as their degree of influence over others who are all embedded in sociopolitical networks is proportional to her Katz-Bonacich centrality measure in those networks (Armengol,, et. al., 2009). This means that an individual's network location determines influence on his and her surrounding network structure. Individuals are affected by the spatial property of their social networks and their peers. People are not simply persuaded by information they receive directly from the media. They perceive their surroundings at various levels and at different intensities. A *social voter* is an individual whose opinions are largely shaped by opinions of people in his/her sphere of social influence with small, endogenous preferences leaning towards public welfare. People relay and sustain opinions of their local environment. We are building agent-based models to explore a range network effects on individual reasoning.

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