# A Dynamic Network Interdiction Problem

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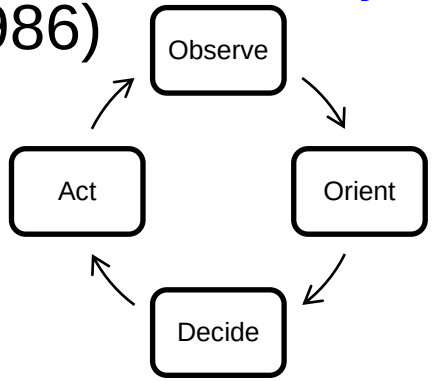
# Motivation

- Network interdiction is not a static process. Decisions are revisited and strategies evolve.

- Motivated study:

  Network interdiction with consideration of the temporal domain for strategic decisions by the interdictor and evader, considering multiple resources and different relative lengths for the respective implementation cycles, a.k.a., OODA loops (Boyd, 1986)




OODA loop*

* Similar to the PDCA/Schwartz/Deming models for business

# Math Programming Model

- Objective: minimize the maximum regret, consisting of weighted measures of:
  - Interdictor costs (resource procurement, deployment, employment)
  - Evader penalties due to physically interdicted flow
  - Evader net flow

- Assumptions
  - Known evader source and terminus nodes
  - Multiple resources and partial arc interdictions
  - Evader and interdictor, respectively, can change their strategies at a fixed period, albeit not necessarily of the same length
  - Preemptively weight flow over penalties to ensure maximum flow by evader

# Solution Procedures - Reducing the Challenges

Minimax MINLP with complicating constraints between periods

1. Dualize inner maximization problem
2. Preemptively weight flow
3. Scale objective function

Well-scaled nonconvex MINLP* to minimize

* Nonlinearity due to binlinear terms in the objective function

# Solution Procedures Examined

A. Direct solution via commercial solver BARON

B. Alternating heuristic
   1. Fix one set of bilinear terms to feasible non-zero values and solve resulting MIP (#1) using CPLEX.
   2. Using incumbent solution, fix values for second set of bilinear terms and solve resulting MIP (#2) using CPLEX.
   3. Using updated incumbent solution, fix values for first set of bilinear terms and solve resulting MIP (#1) using CPLEX. If insufficient improvement results, terminated with the prescribed solution.
   4. Repeat Steps 2 and 3 until insufficient improvement results at either step, and terminate with the prescribed solution.

# Stability Analysis via BARON Commercial Solver

- Examined stability and convergence of strategies over:
  - Two types of problem structures (using very small instances)
    - Unique optimal solution for minimax net flow
    - Alternative optimal solutions for minimax net flow
  - Three cases of relative decision cycle lengths
  - Three lengths of time

| For an instance with: | An optimal solution… | BARON can attain… |
|---|---|---|
| Unique minimax net flow solution | Converges to a stable equilibrium | An optimal solution within 1500 CPU seconds |
| Alternative optimal minimax net flow solutions | Converges to a region of bounded oscillation | Near-optimal solutions for moderate time horizons (within 5 CPU hrs) |

# Alternating Heuristic Performance

- Compared to BARON performance for larger-sized instances
  - Digraph over matrix of (*m* x *n*) nodes between *s* and *t* (Israeli and Wood, 2002)
  - $\Gamma$=12 periods, with three relative decision cycle lengths for the interdictor and evader
  - Two resource problem

| (m, n) | $(\gamma^I,\gamma^E)=(2,2)$ | | $(\gamma^I,\gamma^E)=(2,3)$ | | $(\gamma^I,\gamma^E)=(3,2)$ | |
|---|---|---|---|---|---|---|
| | Imp (%) | CPU Sec | Imp (%) | CPU Sec | Imp (%) | CPU Sec |
| (5, 5) | 69.52 | 1.434 | 69.46 | 1.246 | 67.98 | 0.998 |
| (5, 10) | 69.68 | 1.761 | 71.15 | 1.81 | 69.52 | 1.779 |
| (10, 5) | 71.33 | 1.543 | 71.72 | 1.672 | 66.77 | 1.214 |
| (10, 10) | 72.48 | 8.010 | 65.91 | 8.521 | 62.10 | 5.553 |

- Heuristic Performance Summary
  - Objective function values: average of 68.97% lower than reported by BARON upon its premature termination at 1800 CPU seconds
  - Computational effort: average of 2.962 CPU seconds

# Conclusions and Recommendations

- Conclusions
  - Incorporation & examination of a psychological decision-making framework within an operations research model
  - Application of pre-emptive weighting within a non-preemptive formulation
  - Examination of stability, convergence, & oscillation of strategies
  - Development of a heuristic procedure that outperforms an exact algorithm via commercial software

- For future research
  - Customized algorithm using MIP-relaxations via a Reformulation Linearization Technique within a branch-and-bound framework