# Interdicting Networks to Competitively Minimize Evasion with Synergy Between Applied Resources

Brian J. Lunday[1] Hanif D. Sherali[2]

[1]Department of Mathematical Sciences,
United States Military Academy at West Point

[2]Grado Department of Industrial and Systems Engineering,
Virginia Polytechnic Institute and State University

INFORMS Annual Conference
November 10, 2010

Acknowledgements: Dr. Nick Sahinidis of the Sahinidis-Optimization Group at CMU
Omar Nelson Bradley Officer Research Fellowship in Mathematics

# Motivation from Applied Problems

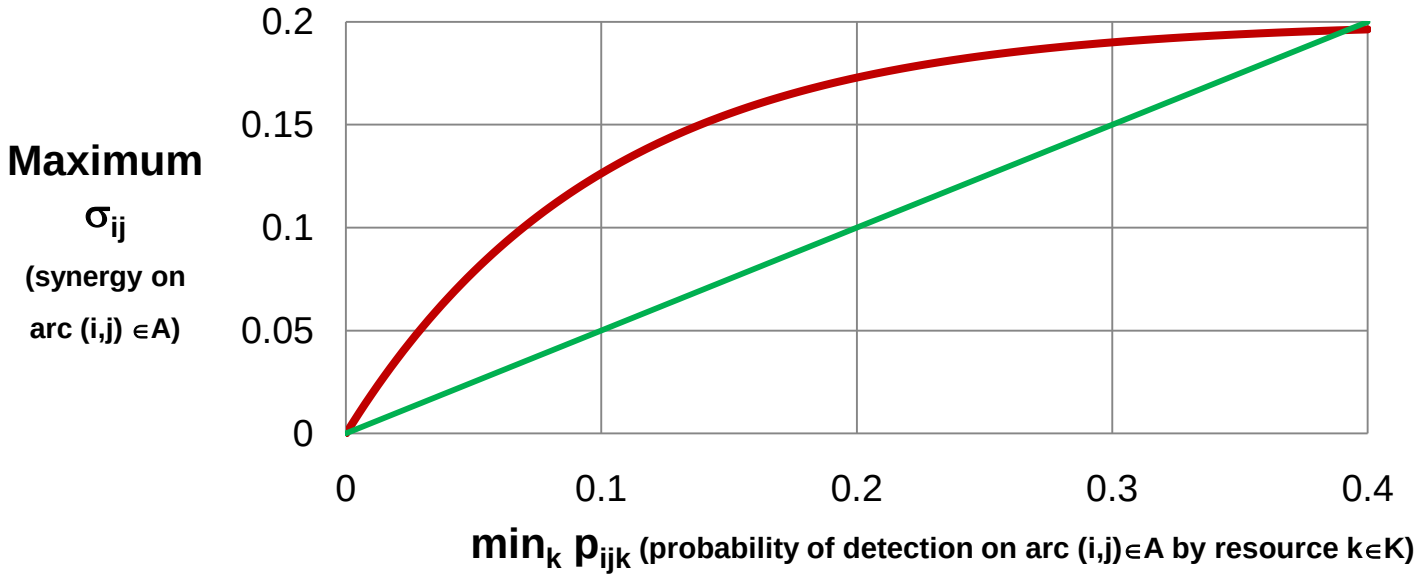
- Network interdiction
  - During a military deployment by one author to Iraq, US and Iraqi Security Forces periodically conducted missions to physically interdict (detect) a single evader transiting a network:
    1. Kidnapping and transport of a reporter for the Christian Science Monitor, Jan 2006
    2. Capture and transport of a US Soldier, Oct/Nov 2006
    3. *Transport of detainees between local and long-term detention facilities
    4. *Transport of a detainee for testimony

- Synergy
  - US and Iraqi forces had different capabilities that manifest superadditive synergy when utilized in local, combined operations

- Resource utilization
  - Overt is highly visible, but allows for synergy
  - Covert has a reduced signature, but often forgoes synergistic employment

# Purpose and Scope of Research

- Examine the competitive probability of evasion problem (CPEP) with multiple resources, considering (1) no synergy, (2) linear synergy, (3) concave nonlinear synergy, and (4) a combined strategy with subsequent overt and covert resource deployments



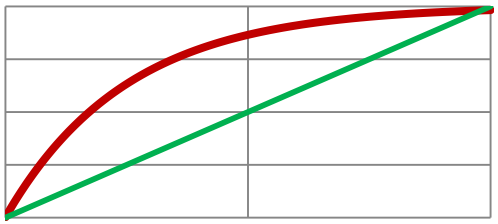

- Develop effective solution procedures
- Conduct theoretical and empirical comparison of strategies and develop insights, as appropriate

# Overt Deployment Strategies and Models

- Objective: minimize the maximum probability of evasion
- Assumptions
  - Known source and terminus nodes
  - Evader will send one unit of flow across the network
  - Resource types ($k \in K$) constrained by availability ($R_k$)
  - Probabilities of detection on arcs are independent and cumulative
- Model
  - $r_{ijk}$: the amount of resource $k$ allocated to overtly detect an evader on arc $(i,j)$, based on the cost of certain detection ($c_{ijk}$)
  - Given values for $r_{ijk}$, $\forall\ (i,j) \in A$, $k \in K$, the additional probability of detection due to to synergy takes the form

Problem CPEP: $\sigma_{ij} = 0, \forall (i, j) \in A$

Problem CPEP-LS: $\sigma_{ij} \le f\left(\frac{r_{ijk}}{c_{ijk}}\right), \forall (i, j) \in A, k \in K$



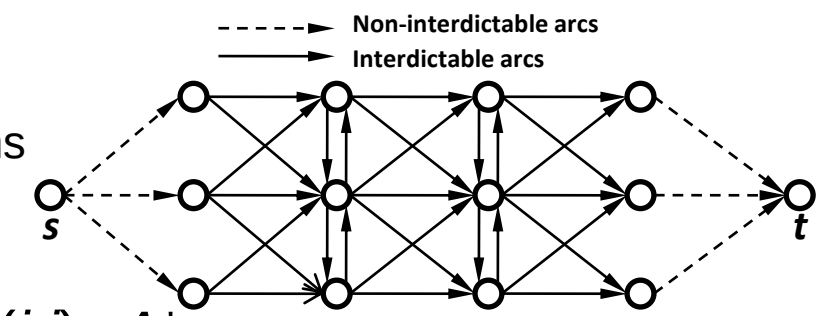
Problem CPEP-NLS: $\sigma_{ij} \le g\left[1 - e^{-10\left(r_{ijk}/c_{ijk}\right)}\right], \forall (i, j) \in A, k \in K$

# Covert Deployment Strategies and Models

- Objective: minimize the maximum probability of evasion
- Given
  - Solution to Problem CPEP using subset of resource types to attain probabilities of evasion ($p_{ij}$*) on arc (*i,j*)
  - Subset of resource types for covert deployment
  - $r_{ijk}$: the amount of resource *k* allocated to covertly detect an evader on arc (*i,j*), based on the cost of certain detection ($c_{ijk}$)
- Models
  - Post-CPEP$_1$ (Revealed path)
    - Motivation: *directed intelligence collection* and *tactical patience*
    - Evader reveals the selection of a path through network after deployment of overt resources, allowing concentration of covert resources
  - Post-CPEP (Unrevealed path)
    - Evader is assumed to traverse any one of the paths having a maximal probability of evasion in the optimal solution to Problem CPEP

# Solution Procedure Development

- CPEP, CPEP-LS, CPEP-NLS, Post-CPEP
  - Resulting objective function $\min_{r,p,\sigma} \max_{x} \prod_{(i,j)\in A} \left(1 - x_{ij} + p_{ij} x_{ij}\right)$
  - Path-restricted reformulations $\min_{r,p} P; s.t. P \geq \prod_{(i,j)\in l} p_{ij}, l \in L$
  - Formulations allow commercial solver application, although cumbersome, given *our network topography*
    - 3 x 3 → 49 directed acyclic paths
    - 4 x 4 → 1000 directed acyclic paths
    - 5 x 5 → 28,561 directed acyclic paths
    - *m* x *n* → |L| ≤ $(3m)^{(n-1)}$ paths




- Post-$CPEP_1$ with path denoted by $(i,j) \in A^*$
  - **Proposition**: One covert resource type – optimal solution allocates resource to the most critical arc, defined by

$$(i, j) = \arg\min_{(i,j)\in A^*} \left\{ p_{ij}^* c_{ij} \right\}$$

  - More than one covert resource type
    - Requires optimization of model

# Covert Strategy Testing and Comparison

- Testing parameters
  - 30 instances on a 3x3 network
  - 2 resources: 1 overtly deployed and 1 deployed based on the strategy
  - Detection costs correlated across resource types
  - Solved with BARON

| Covert Strategy Modeled | Revealed Path | Integer-restricted Resource Applications | Avg. Relative Gap (%) from Optimal Solns to: | | |
|---|---|---|---|---|---|
| | | | CPEP | CPEP-LS | CPEP-NLS |
| 1 | | | 6.32E-07 | 11.32 | 25.94 |
| 2 | | X | -59.13 | -51.71 | -48.16 |
| 3 | X | | -92.68 | -89.09 | -91.88 |
| 4 | X | X | -90.80 | -90.24 | -90.08 |

- Relative optimality gaps
  - Increased with greater relative availability of covert resource
  - Increased in magnitude with greater total of both resources
- Strategy-specific computational results:
  1. Never superior to an overt strategy
  2. Average 61.85% reduction in the number of paths to be considered for covert resource application
  3. Never inferior to an overt strategy
  4. Never inferior to an overt strategy

# Revealed Optimal Evader Path Theoretical Results

For non-integer and integer-restricted resource applications:

- **Proposition**: In the presence of synergy, an overt strategy can yield a strictly better solution than a covert strategy based on a revealed evader path.

- **Proposition**: Without synergy, an overt strategy is never better than a covert strategy on a network having disjoint paths, each exhibiting the Monotone Allocation Property (MAP)
  - MAP: probability of evasion on each path is monotonically decreasing with respect to the total resources assigned.
  - Some demonstrated conditions for MAP:

    1) $c_{ij} = c_{ijk}, \forall (i, j) \in A, k \in K$

    2) $c_{ijk} = \alpha_k, \forall (i, j) \in A, k \in K$

# Conclusions and Recommendations

- Results
  - Novel formulations to represent alternative interdictor resource deployment strategies
  - Identification of conditions preferable for deployment of resources covertly

- Future Research
  - Sociological research on the forms of superadditive synergy between groups/agencies
  - Consider the validity of alternate probability-resource and synergy-resource relationships that affect a convex program for CPEP and its variants
  - Allow for ‘fuzzy’ perceptions of interdicted network states by evader

# References


Bayrak, H., and Baily, M. (2008). Shortest Path Network Interdiction with Asymmetric Information. *Networks*, **52** (3), 133–140.

Brown, S. S. (1980). Optimal Search for a Moving Target in Discrete Time and Space. *Operations Research*, **28** (6), 1275–1289.

Brown, G., Carlyle, M., Salmerón, J., and Wood, R. K. (2006). Defending Critical Infrastructure. *Interfaces*, **36** (6), 530–544.

Cormican, K. J. (1995). Computational Methods for Deterministic and Stochastic Network Interdiction Problems. Master's Thesis, US Naval Postgraduate School, Monterey, CA.

Fulkerson, D. R., and Harding, G. C. (1977). Maximizing the Minimum Source-Sink Path Subject to a Budget Constraint. *Mathematical Programming*, **13** (1), 116–118.

Israeli, E., and Wood, R. K. (2002). Shortest-Path Network Interdiction. *Networks*, **40** (2), 97–111.

Koopman, B. O. (1979). Search and Its Optimization. *The American Mathematical Monthly*, **86**, 527–540.

Lunday, B. J., and Sherali, H. D. (2009). Network Flow Interdiction Models and Algorithms with Resource Synergy Considerations. Manuscript, Grado Department of Industrial and Systems Engineering, Virginia Polytechnic Institute and State University, Blacksburg, VA.

Napier, R. W., and Gershenfeld, M. K. (1993). *Groups: Theory and Experiences*. Boston, MA: Houghton Mifflin Company.

Osborne, M. J. (2004). *An Introduction to Game Theory*. New York, NY: Oxford University Press.

Pan, F., Charlton, W. S., and Morton, D. P. (2003). A Stochastic Program for Interdicting Smuggled Nuclear Material. In D. L. Woodruff (Ed.), *Network Interdiction and Stochastic Integer Programming* (pp. 1–19). Norwell, MA: Kluwer Academic Publishers.

von Eye, A., Schuster, C., and Rogers, W. M. (1998). Modelling Synergy using Manifest Categorical Variables. *International Journal of Behavioral Development*, **22** (3), 537–557.

Washburn, A., and Wood, R. K. (1995). Two-Person Zero-Sum Games for Network Interdiction. *Operations Research*, **43** (2), 243—251.