Equitable Apportionment of Railcars within a Pooling Agreement for Shipping Automobiles

Brian J. Lunday[1] Hanif D. Sherali[2]

[1]Department of Mathematical Sciences,
United States Military Academy at West Point

[2]Grado Department of Industrial and Systems Engineering,
Virginia Polytechnic Institute and State University

INFORMS Annual Conference
November 8, 2010

Acknowledgements: Dr. Moncef Sellami and TTX Reload Group
Omar Nelson Bradley Research Fellowship in Mathematics

Railcar Allocation – Problem Statement

Given: The total fleet size to be jointly utilized over the next year by the consortium of automobile manufacturers and railroads participating in the pooling agreement, as determined by a strategic planning (i.e., fleet sizing) process that considers static and dynamic demand profiles.

Find: An equitable allocation of railcars to each automobile manufacturer (shipper) from the given total fleet size, with a subsequent equitable allocation of railcar acquisition responsibility for each railroad.

Railcar Allocation Problem: Fleet Sizing Background

- Spectrum of approaches
  - Reverse routing on OD pairs
  - Solve transportation problem for each shipper and carrier to meet demand ➔ possible reduction in mileage (time) and fleet size
  - Pooled resources across automobile shippers and carriers ➔ greater reduction in time and fleet size

- Deregulation
  - Motor Carrier Act of 1980 (Public Law 96-296)
  - Railroad Transportation Policy Act of 1979 (Public Law 96-448),

- TTX Reload Group established in 1982
  - Coordinates operations of 9 shippers, 17 carriers, and 59,000 auto railcars
  - Saves over 1B miles/year of empty railcar movement over reverse routing
  - Employs static and dynamic fleet sizing, followed by simple shipper and carrier allocation schemes, enabling solutions within five minutes

Railcar Allocation Problem: Fleet Sizing Process

- Static Fleet Sizing (Avg. the peak 3 monthly forecasted volumes)

$$F_S = \sum_{(i,j)\in OD}\sum t_{ij}Q_{ij} + \sum_{(p,q)\in A}\sum \tau_{pq}\bar{x}_{pq}$$

(Loaded railcar days) (Empty railcar days)

(Q_{ij} values are adjusted based on historical forecast accuracy factors for each shipper)

- Dynamic Fleet Sizing (Avg. the 3 highest monthly sizings)

$$F_D \quad \Rightarrow \quad DF = F_D / F_S$$

$$FS = \sum_{(i,j)\in OD}\sum FS(i,j) = \sum_{(i,j)\in OD}\sum (DF)Q_{ij}\left(t_{ij} + \phi_i + t_{ij}^q\right)\beta$$

(Adjusted for dynamic factor, queuing, bad order factor)

$$\phi_i = \frac{\sum_{p:(p,i)\in A}\tau_{pi}\bar{x}_{pi}}{\sum_{p:(p,i)\in A}\bar{x}_{pi}}$$

(Weighted average empty return time to origin *i*)

Current Railcar Allocation Scheme

- Shipper Allocations $(k \in K)$

  (in proportion to the weighted average empty return time to the origin(s))

$$FS^{PAS}(k) = \sum_{(i,j)\in OD_k}\sum FS(i,j),\ \forall\, k \in K$$

- Carrier Allocations $(r \in R)$

  (in proportion to the loaded railcar-days of business conducted with each railroad)

$$FS_r^{PAS} = \sum_{k\in K} FS_{rk}^{PAS},\ \forall\, r \in R$$

$$FS_{rk}^{PAS} = FS^{PAS}(k)\left(\frac{L_{rk}}{\sum_{\rho\in R} L_{\rho k}}\right),\ \forall\, r \in R,\ k \in K$$

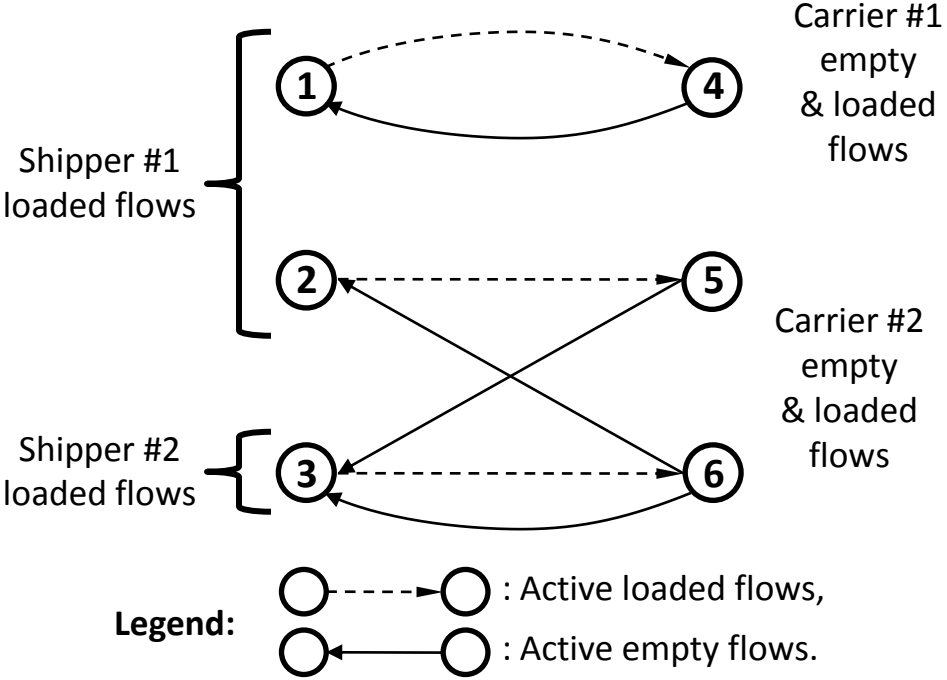
(proportional weighting)

$$L_{rk} = \sum_{(i,j)\in OD_k}\sum f_{ijr}^{k} Q_{ij} t_{ij},\ \forall\, r \in R,\ k \in K$$

(fractional responsibility)

Railcar Allocation Problem: Motivation (1 of 2)

- Neglects separability of network components into disjoint subproblems
  - Allows characteristics of a component to inequitably affect allocations for others
  - Each component should be addressed separately



<u>Fleet size for OD-Pairs</u>

FS(1,4)=10
FS(2,5)=20
FS(3,6)=10

<u>Shipper Allocations</u>

$FS^{PAS}(1)=30$
$FS^{PAS}(2)=10$

<u>Loaded carrier-shipper railcar days</u>

$L_{11}=5$
$L_{21}=5$
$L_{12}=0$
$L_{22}=5$

<u>Carrier-to-shipper Allocations</u>

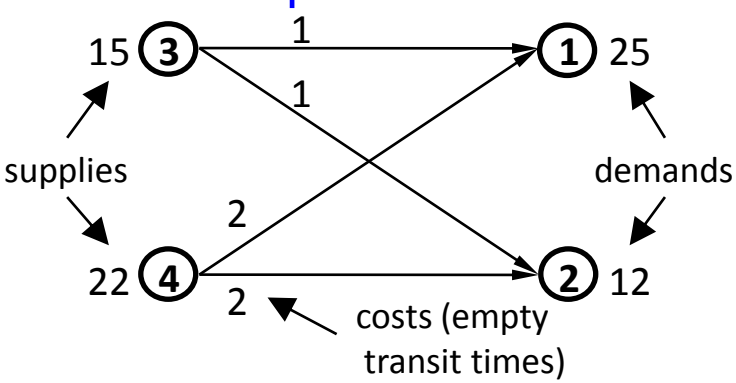
$FS^{PAS}_{11}=15$
$FS^{PAS}_{21}=15$
$FS^{PAS}_{12}=0$
$FS^{PAS}_{22}=10$

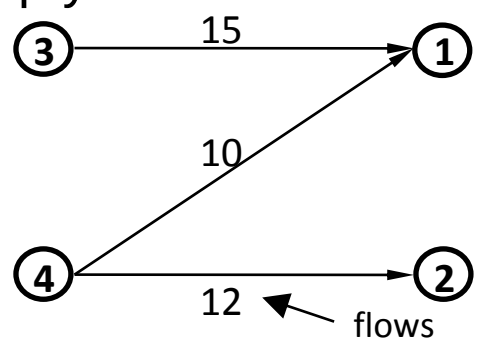
<u>Carrier Allocations</u>

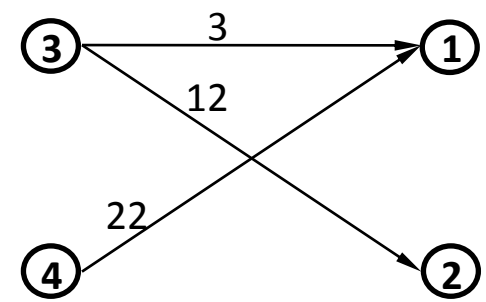
$FS^{PAS}_{1}=15$
$FS^{PAS}_{2}=25$

Railcar Allocation Problem: Motivation (2 of 2)

- If alternative optimal solutions for empty railcar flow exist
  - Shipper allocation via average nodal empty transit times may affect different (i.e., inequitable) shipper allocations and subsequent carrier allocations
  - Must utilize allocation method that is independent of alternative optimal solutions for empty railcar flow







(i,j)	(1,3)	(1,4)	(2,3)	(2,4)
Q_{ij}	15	10	--	12
t_{ij}	1	2	-	1
(f^L_{ij1}, f^L_{ij2})	(1,0)	(0,1)	(--,--)	(0.5,0.5)
τ_{ji}	1	2	1	2
β=DF=1				

<u>Flow Solution #1</u>

ϕ_1=1.4	FS=106
ϕ_2=2	$FS^{PAS}(1)$=70
FS(1,3)=36	$FS^{PAS}(2)$=36
FS(1,4)=34	FS^{PAS}_1=48
FS(2,4)=36	FS^{PAS}_2=58

<u>Flow Solution #2</u>

ϕ_1=1.88	FS=106
ϕ_2=1	$FS^{PAS}(1)$=82
FS(1,3)=43.2	$FS^{PAS}(2)$=24
FS(1,4)=38.8	FS^{PAS}_1=47.1
FS(2,4)=24	FS^{PAS}_2=58.9

Alternative Allocation Schemes Examined

Schemes	To Shippers	Governing Equations
Current	PAS - Loaded Transit Times	$FS^{PAS}(k) = \sum\sum_{(i,j)\in OD_k} FS(i,j),\ \forall\, k \in K$
Proposed	SA1 - Loaded Transit Plus Queuing Times	$FS^{SA1}(k) = FS\left[\frac{L(k)}{\sum_{k'\in K} L(k')}\right],\ where\ \ L(k) = \sum\sum_{(i,j)\in OD_k} Q_{ij}\left(t_{ij} + t_{ij}^{q}\right)$
	SA2 - Marginal Cost Analysis #1	$FS^{SA2}(k) = FS\left[\frac{L'(k)}{\sum_{k'\in K} L'(k')}\right],\ where\ \ L'(k) = \sum\sum_{(i,j)\in OD_k} Q_{ij}\left(t_{ij} + t_{ij}^{q}\right) + \left(\frac{\psi_k}{\sum_{k'\in K}\psi_{k'}}\right)\bar{\upsilon}$ $\psi_k = \bar{\upsilon} - \bar{\upsilon}_k$
	SA3 - Marginal Cost Analysis #2	$FS^{SA3}(k) = FS\left[\frac{L''(k)}{\sum_{k'\in K} L''(k')}\right],\ where\ \ L''(k) = \sum\sum_{(i,j)\in OD_k} Q_{ij}\left(t_{ij} + \phi_{ij}^{''} + t_{ij}^{q}\right)$ $\phi_i^{''} = \bar{\upsilon}\left[\frac{\psi'_i}{\sum_{i'\in I}\psi'_{i'}}\right]\frac{1}{\sum_{j:(i,j)\in OD} Q_{ij}}$ $\psi_i = \bar{\upsilon} - \bar{\upsilon}_i$
	SA4 - Shapley Value Allocations	$FS^{SA4}(k) = \sum_{S\subseteq K: k\in S} \frac{(\lvert S\rvert - 1)!(K - \lvert S\rvert)!}{K!}\left[V(S) - V(S\setminus\{k\})\right],\ where\ V(S) = FS_S$

*Marginal cost examined for each shipper as an entity (SA2), or via its node-wise contributions (SA3)

Alternative Shipper Allocation Schemes Examined

Schemes	To Carriers	Governing Equations
Current	RA1 - Loaded railcar movements	$FS_r^{PAS} = \sum_{k \in K} FS_{rk}^{PAS},\ \forall\, r \in R$ $FS_{rk}^{PAS} = FS^{PAS}(k)\left(\frac{L_{rk}}{\sum_{\rho \in R} L_{\rho k}}\right),\ \forall\, r \in R,\ k \in K$ $L_{rk} = \sum\sum_{(i,j) \in OD_k} f_{ijr}^{k} Q_{ij} t_{ij},\ \forall\, r \in R,\ k \in K$
Proposed	RA2 - Total Capital Plus Operating Costs	$Minimize\left\{\sum_{r \in R}\left(f_r - \overline{f}_r\right)^2 : \sum_{r \in R} f_r = FS;\ f_r \geq 0,\ \forall\, r \in R\right\}$ $\overline{f}_r = \left(\frac{\sigma \sum_{k \in K} L_{rk} - \xi_r}{c}\right),\ \forall\, r \in R,\ where\ \sigma = \left(\frac{c(FS) + \sum_{r \in R} \xi_r}{\sum_{r \in R}\sum_{k \in K} L_{rk}}\right)$

(railcar cost per annum) (empty railcar movement cost per annum)

- **Proposition**: Present carrier allocation scheme (RA1) is equivalent to a game-theoretical approach utilizing Shapley value allocations
- Cost-based carrier allocations (RA2) are independent of shipper allocations, and based on $\overline{f}_r$–values that are directly proportional to the respective loaded rail-car days of business

Allocation Schemes – Coding and Implementation

- Considered 10 combinations of current and proposed methodologies
- Coded in C++ to call CPLEX 11.1 using Concert Technology to solve the initial fleet sizing problem, and the additional K, m, or 2^K-1 related problems, as appropriate
- Generated 20 test instances based on parametric data from the TTX Reload Group
- Initial results (one instance) for method combinations
  - 20 shippers, 7 carriers, 60 origin nodes, 86 destination nodes
  - Significant variability among shipper allocations

    $s \in [1.98, 49.51]$ for $\overline{x} \in [294.1, 2339.2]$
  - Lesser variability among carrier allocations

    $s \in [0.80, 11.95]$ for $\overline{x} \in [2008.1, 3423.9]$

Relative Changes from Benchmark Allocation Methods (PAS and PAS-RA1) over 30 Instances

- Relative (%) Changes to Shipper Allocations from PAS

	Mean Abs. Dev.	Std. Dev.	Max.	Min.	Mean Pos.	Mean Neg.	Inc:Dec Ratio
SA1	2.459	2.997	5.209	-4.805	2.519	-2.444	1.119
SA2	2.351	2.880	4.904	-4.737	2.424	-2.362	1.142
SA3	2.349	2.884	4.917	-4.763	2.399	-2.392	1.164
SA4	1.455	1.891	4.564	-2.378	1.754	-1.095	1.389

- Relative (%) Changes to Carrier Allocations from PAS-RA1

	Mean Abs. Dev.	Std. Dev.	Max.	Min.	Mean Pos.	Mean Neg.	Inc:Dec Ratio
SA1-RA1	0.0179	0.024	0.035	-0.036	0.019	-0.019	1.074
SA2-RA1	0.0176	0.023	0.033	-0.036	0.019	-0.018	1.011
SA3-RA1	0.0173	0.023	0.033	-0.036	0.018	-0.018	1.016
SA4-RA1	0.0114	0.015	0.023	-0.023	0.012	-0.013	1.225
PAS-RA2, SAα-RA2, α=1,…,4	0.5655	0.703	0.625	-1.206	0.471	-0.655	0.859

- Testing conclusions
  - Greater variability among shippers' alternative allocations than those for carriers
  - Inc:Dec ratios provide empirical evidence that a redistribution of capital is in order
  - Even SA4 was not too computationally expensive, requiring an average of 41.0 CPU minutes to solve the instances sized similar to current environment.

Conclusions and Recommendations

- Results
  - Demonstrated need to alter existing aggregate approach and replace current shipper allocation method to prevent inequities
  - Reinforced value of current carrier allocation method
  - Demonstrated several philosophically equitable alternatives

- Future Research
  - Increase granularity of transportation costs with respect to specific cargoes

References (1 of 2)

Altman, E., Jimenez, T., Koole, G., 2001. On Optimal Call Admission Control in a Resource-sharing System. *IEEE Transactions on Communications* **49** (9), 1659–1668.

Assad, A.A., 1980. Models for Rail Transportation. *Transportation Research Part A* **14** (3), 205–220.

Beaujon, G. J., Turnquist, M. A., 1991. A Model for Fleet Sizing and Vehicle Allocation. Transportation Science **25** (1), 19–45.

Bitran, G., Hax, A., 1976. On the Solution of Convex Knapsack Problems with Bounded Variables. In: *Proceedings of the 9th International Symposium on Mathematical Programming* , North-Holland Publishing Company, Budapest, 357–367.

Cheng, Y., Lin, W., 2005. Comparison of Methods for Allocating Costs of Empty Railcar Movements in a Railcar Pooling System. *Transportation Research Record* **1916**, 88-–95.

Cordeau J. F., Toth P., Vigo D., 1998. A Survey of Optimization Models for Train Routing and Scheduling. *Transportation Science* **32** (4), 380–404.

Davis, P. M., 1984. The Cost of Empty Rail Car Supply: A Method of Allocating Empty Costs to Loaded Trips. *Transportation Research Record* **966** , 8–14.

Dejax, P. J., Crainic, T. G., 1987. A Review of Empty Flows and Fleet Management Models in Freight Transportation. *Transportation Science* **21** (4), 227–247.

Driessen, T., 1988. *Cooperative Games, Solutions, and Applications* , Kluwer Academic Publishers, Dordrecht, The Netherlands.

Ergun, O., Kuyzu, G., Savelsbergh, M., 2007. Reducing Truckload Transportation Costs Through Collaboration. *Transportation Science* **41** (2), 206–221.

Glickman, T. S., Sherali, H.D., 1985. Large-scale Network Distribution of Pooled Empty Freight Cars Over Time, with Limited Substitution and Equitable Benefits. *Transportation Research Part B* **19** (2), 85–94.

Kikuchi, S., 1985. Empty Freight Car Dispatching Model Under Freight Car Pool Concept. *Transportation Research Part B* **19** (3), 169–185.

Örmeci, E. L., Burnetas, A., 2005. Dynamic Admission Control for Loss Systems with Batch Arrivals. *Advances in Applied Probability* **37** (4), 915–937.

Papier, F., Thonemann, U. W., 2008. Queuing Models for Sizing and Structuring Rental Fleets. *Transportation Science* **42** (3), 302–317.

References (2 of 2)

Roth, A. E., 1988. Introduction to the Shapley Value. In: Roth, A.E. (Ed.), *The Shapley Value - Essays in Honor of Lloyd S. Shapley* , Cambridge University Press, New York, NY, 1–27.

Savin, S. V., Cohen, M. A., Gans, N., Katalan, Z., 2005. Capacity Management in Rental Businesses with Two Customer Bases. *Operations Research* **52** (4), 617–631.

Sayarshad, H. R., Ghoseiri, K., 2009. A Simulated Annealing Approach for the Multi-periodic Rail-car Fleet Sizing Problem. *Computers and Operations Research* **36** (6), 1789–1799.

Shapley, L.S., 1953. A Value for n-person Games. In: Kuhn, H., Tucker, A.W. (Eds.), *Contributions to the Theory of Games*, Vol. 2 , Princeton University Press, Princeton, NJ, 307–317.

Shapley, L.S., 1971. Cores of Convex Games. *International Journal of Game Theory* **1**, 11–26.

Sherali, H. D., Maguire, L. W., 2000. Determining Rail Fleet Sizes for Shipping Automobiles. *Interfaces* **30** (6), 80–90.

Sherali, H. D., Rajan, R., 1986. A Game Theoretic-Mathematical Programming Analysis of Cooperative Phenomena in Oligopolistic Markets. *Operations Research* **34** (5), 683–697.

Sherali, H. D., Shetty, C. M., 1980. On the Generation of Deep Disjunctive Cutting Planes. *Naval Research Logistics Quarterly* **27** (3), 453–475.

Sherali, H. D., Suharko, A. B., 1998. A Tactical Decision Support System for Empty Railcar Management. *Transportation Science* **32** (4), 306–329.

Sherali, H. D., Tuncbilek, C. H., 1997. Static and Dynamic Time-Space Strategic Models and Algorithms for Multilevel Rail-car Fleet Management. *Management Science* **43** (2), 235–250.

Turnquist, M. A., Jordan, W. C., 1986. Fleet Sizing under Production Cycles and Uncertain Travel Times. *Transportation Science* **20** (4), 227–236.

TTX Company, 2010. Reload Operations . [Online] (Updated September 2009) Available at: http://www.ttx.com/capabilities/fleet-operations/reload-operations.aspx [Accessed 28 September 2009].

Žak, J., Redmer, A., Sawicki, P., 2008. Multiple Objective Optimization of the Fleet Sizing Problem for Road Freight Transportation. *Journal of Advanced Transportation* **42** (4), 379–427.