Network Flow Interdiction Models and Algorithms with Resource Synergy Considerations

Brian J. Lunday[1] Hanif D. Sherali[2]

[1]Department of Mathematical Sciences,
United States Military Academy at West Point

[2]Grado Department of Industrial and Systems Engineering,
Virginia Polytechnic Institute and State University

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Motivation from Applied Problems

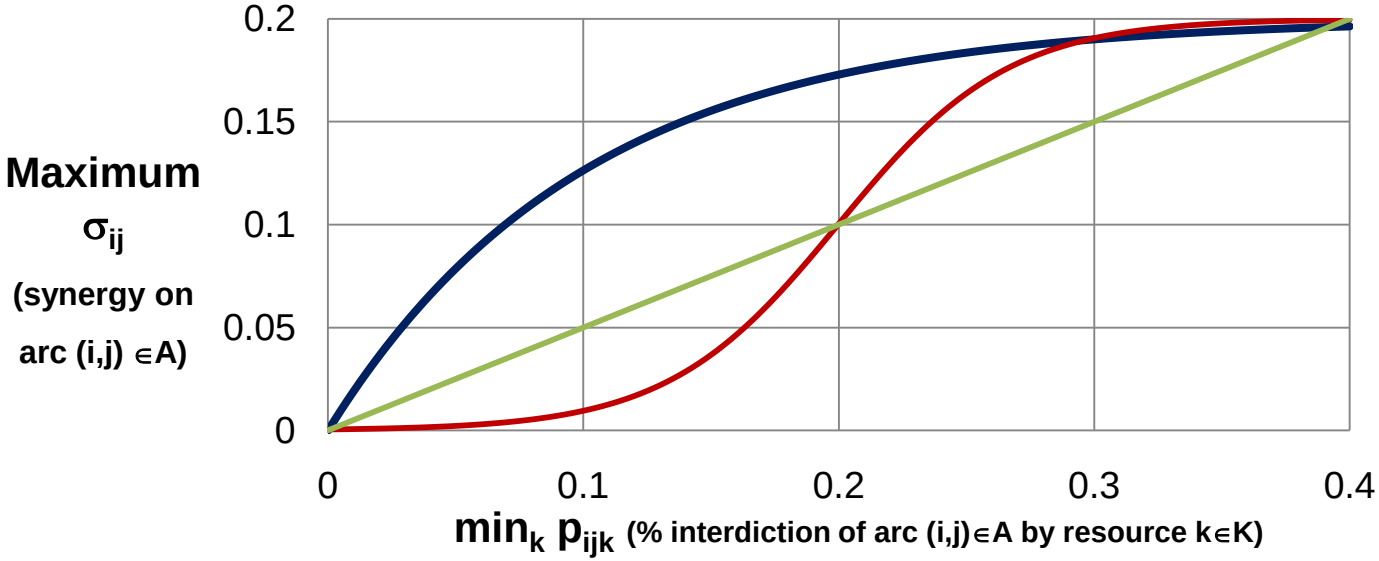
- Network interdictions
  - During a military deployment by one author to Iraq, US and Iraqi Security Forces frequently conducted missions to physically interdict the following net flows:
    1. Import/transport of EFP and IED material to assembly points
    2. Transport of completed EFPs and IEDs to distribution point(s)
    3. Transport of sectarian kidnapping victims, Feb-Nov 2006

- Synergy
  - US and Iraqi forces had different capabilities that manifest superadditive synergy when utilized in local, combined operations

- Other applications
  - Joint, interagency, intergovernmental, and multinational interdiction operations involving resources with some disparate capabilities, deployed to work in concert

Purpose and Scope of Research

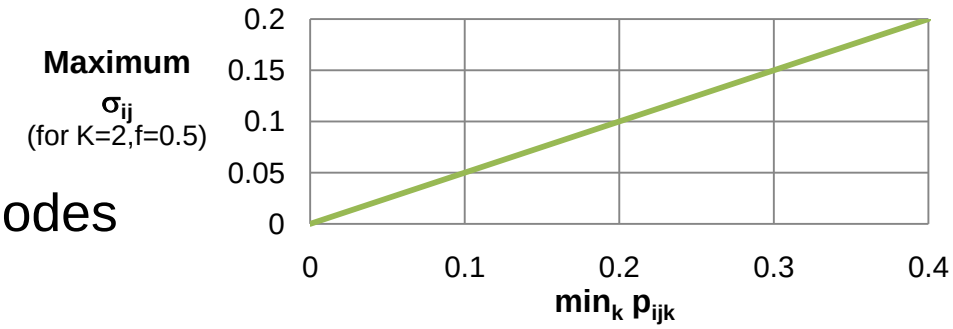
- Examine and improve network resource allocation models via a consideration of microscopic (i.e., 'local') interactions of resources
  - Network interdiction problem with multiple resources (NIP-MR) and synergy, considering (1) linear, (2) concave nonlinear, and (3) general nonlinear superadditive synergy relationships



- Develop effective and efficient algorithms to solve the resulting math programming formulations

NIP-MR with Linear Synergy - Model

- Objective: minimize the maximum flow



- Assumptions
  - Known source and terminus nodes
  - Partial arc interdiction
  - Resource types ($k \in K$) constrained by availability (R_k)

- Model: linear minimax formulation
  - p_{ijk}: the percentage interdiction of the maximum capacity for flow (u_{ij}) on arc (i,j) by resource k based on the cost of complete interdiction (c_{ijk})
  - Given values for p_{ijk}, $\forall$ $(i,j) \in A$, $k \in K$, the additional percentage interdiction due to synergy takes the form

$$\sigma_{ij} \leq f\, p_{ijk}, \forall (i,j) \in A, k \in K$$

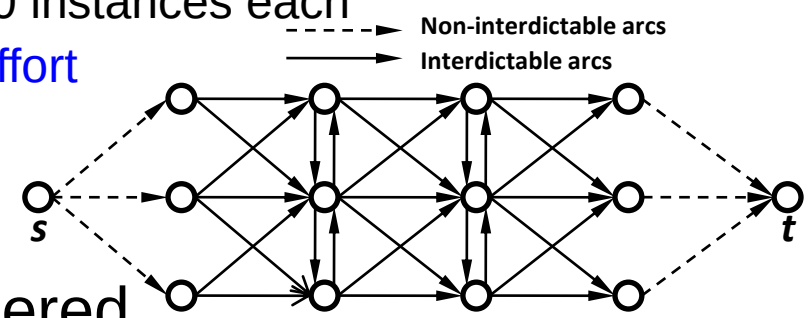
with linear synergy factor $f \in (0,1]$

NIP-MR with Linear Synergy – Solution Procedure

- Direct solution
  - Dualize the inner maximization problem
  - Restrict certain dual variables to binary values
  - → MIP formulation readily solvable via CPLEX

- Alternative solution procedure
  - Reduction by $|A|$ decision variables via Fourier-Motzkin elimination
  - Considered 2-resource problem with $f=0.5$ for solution comparison
    - Set of 15 test problem sizes, with 100 instances each
    - 43.42% reduction in computational effort

Example topography

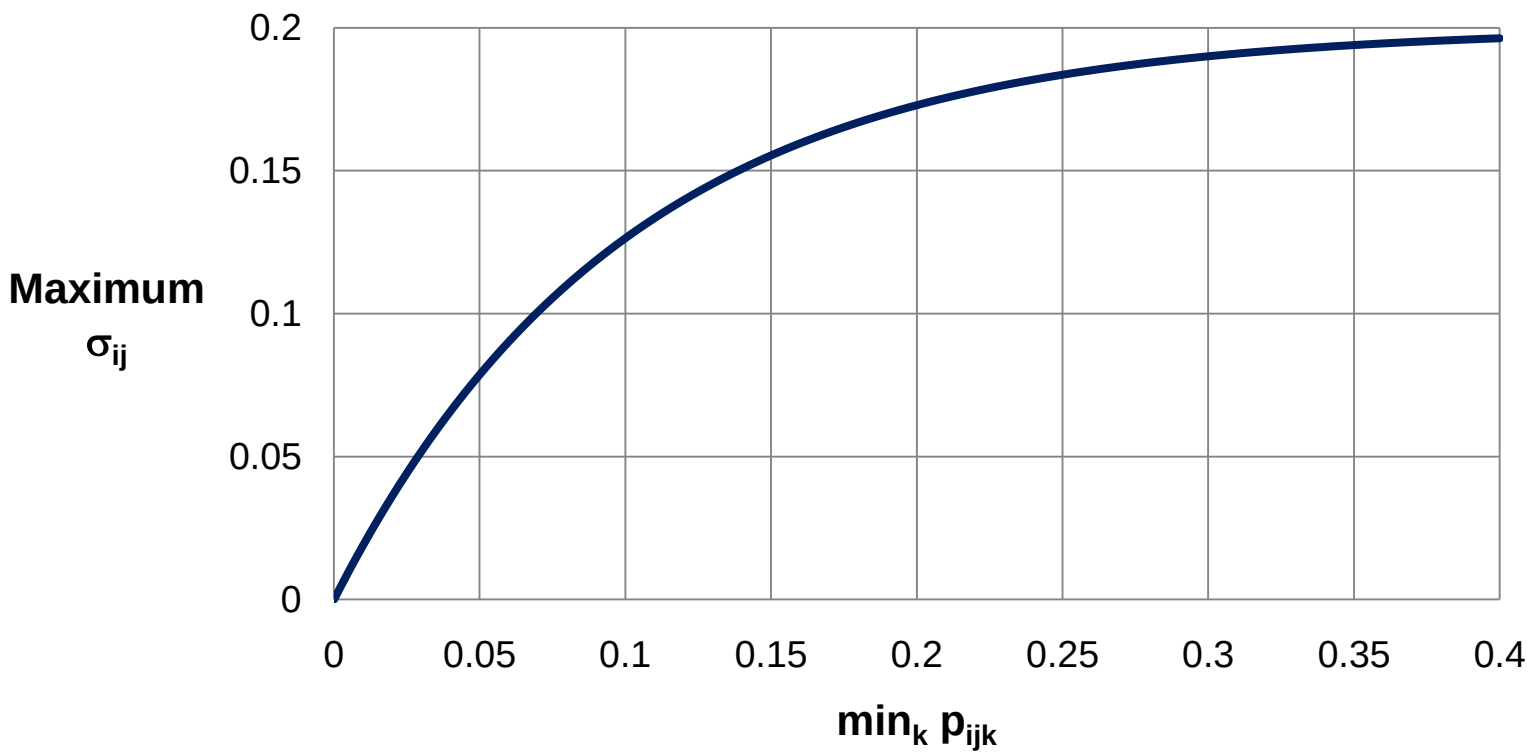


- General impact of synergy considered
  - Average of 5.10% reduction in optimal objective values based on a 0.42% increase in the amount of flow disrupted

NIP-MR with Concave Nonlinear Synergy: Model

- Limiting function for synergy, with proportionality factor *g*

$$\sigma_{ij} \le g\left(1-e^{-10p_{ijk}}\right), \forall (i,j) \in A, k \in K$$



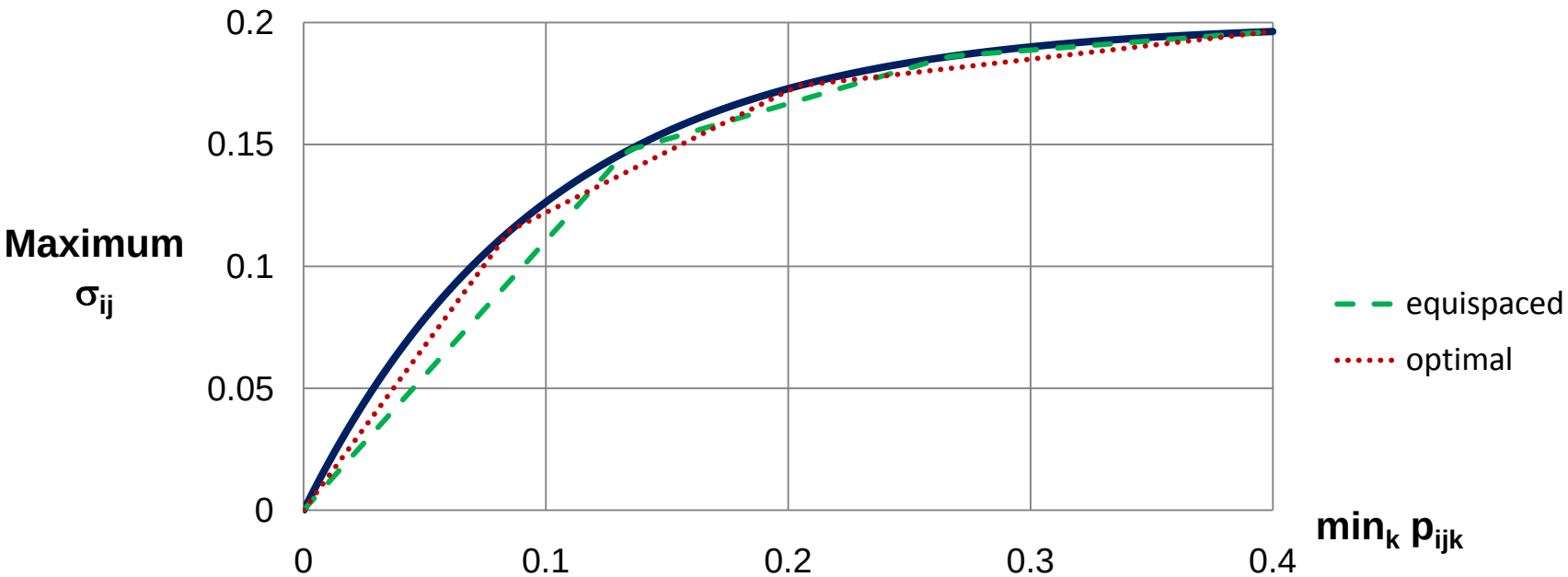
Synergy relationship used for *K*=2, *g*=0.2

- Resulting MINLP formulation has convex relaxation

NIP-MR with Concave Nonlinear Synergy: Algorithm and Heuristics Tested

Algorithm: SBB commercial software with ε=0.01%

1. Inner linearization heuristic
   - Piecewise approximation to nonlinear synergy constraints
     - Employ {2,3,4} segments
     - Equidistant breakpoints and breakpoints located so as to minimize the reduction in the feasible region
   - Solve the restricted problem, fix resulting resource allocations and interdiction percentages, determine actual induced synergy on arcs, and then determine the corresponding maximum flow



2. SBB commercial software with $\varepsilon \in$ {0.1%, 1%}

NIP-MR with Concave Nonlinear Synergy: Comparative Testing Results

- Testing conditions
  - 30 instances each on 15 problems sizes, ranging in size from a 5x5 to a 50x50 intermediary network of nodes
  - Time limit of 1800 CPU seconds
- Results
  - SBB terminated early on the three largest problem sizes, with relative performances as follows:

Problems	Strategy	Equidistant Segments			Optimized Segments			SBB	
		2	3	4	2	3	4	ε=1%	ε=0.1%
1-12	Soln. gap (%)	2.17	2.07	1.88	2.16	1.91	1.95	0.02	0.001
	Rel. time (%)	7.59	10.06	12.39	8.36	11.05	14.06	57.83	66.79
13-15	Soln. gap (%)	-39.03	-38.72	-38.71	-38.80	-38.71	-38.79	-39.52	-39.83
	Rel. time (%)	16.94	22.47	28.03	19.35	26.44	34.29	94.24	100.04

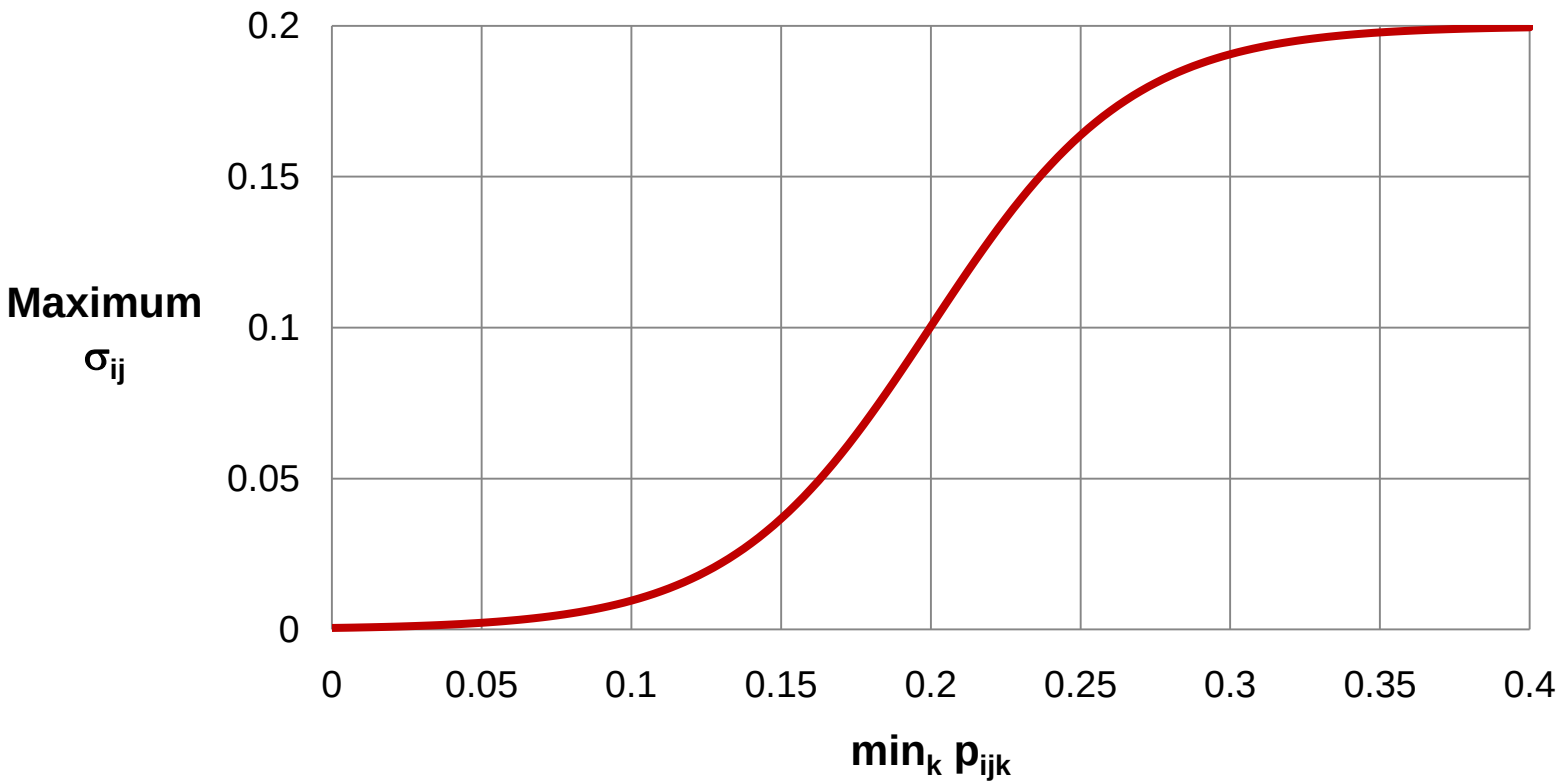
1. Inner linearization heuristic effective on small problems, superior on large problems
   - Strategy with 4 equidistant segments reduces computational effort by a factor of 18; solutions within 3% of optimality on Problems 1-12
2. Use of 'optimized' segment breakpoints not necessarily improvement over equidistant breakpoints, at a 17.03% increase in computational effort
3. SBB with ε={1%,0.1%} were the most effective technique(s), but terminated early for Problems 13-15 on 82.22% and 95.55% of instances, respectively

Additional testing showed BARON Node 0 solutions no more effective than that for SBB
- 0.34% solution gaps and 1755.85% relative times on Problems 1-11
- Unable to attain a feasible solution on Problems 12-15

NIP-MR with General Nonlinear Synergy: Model

- Limiting function for synergy, with proportionality factor *g'*

$$\sigma_{ij} \le \frac{g' e^{30 p_{ijk}}}{2000 + 5 e^{30 p_{ijk}}}, \forall (i, j) \in A, k \in K$$



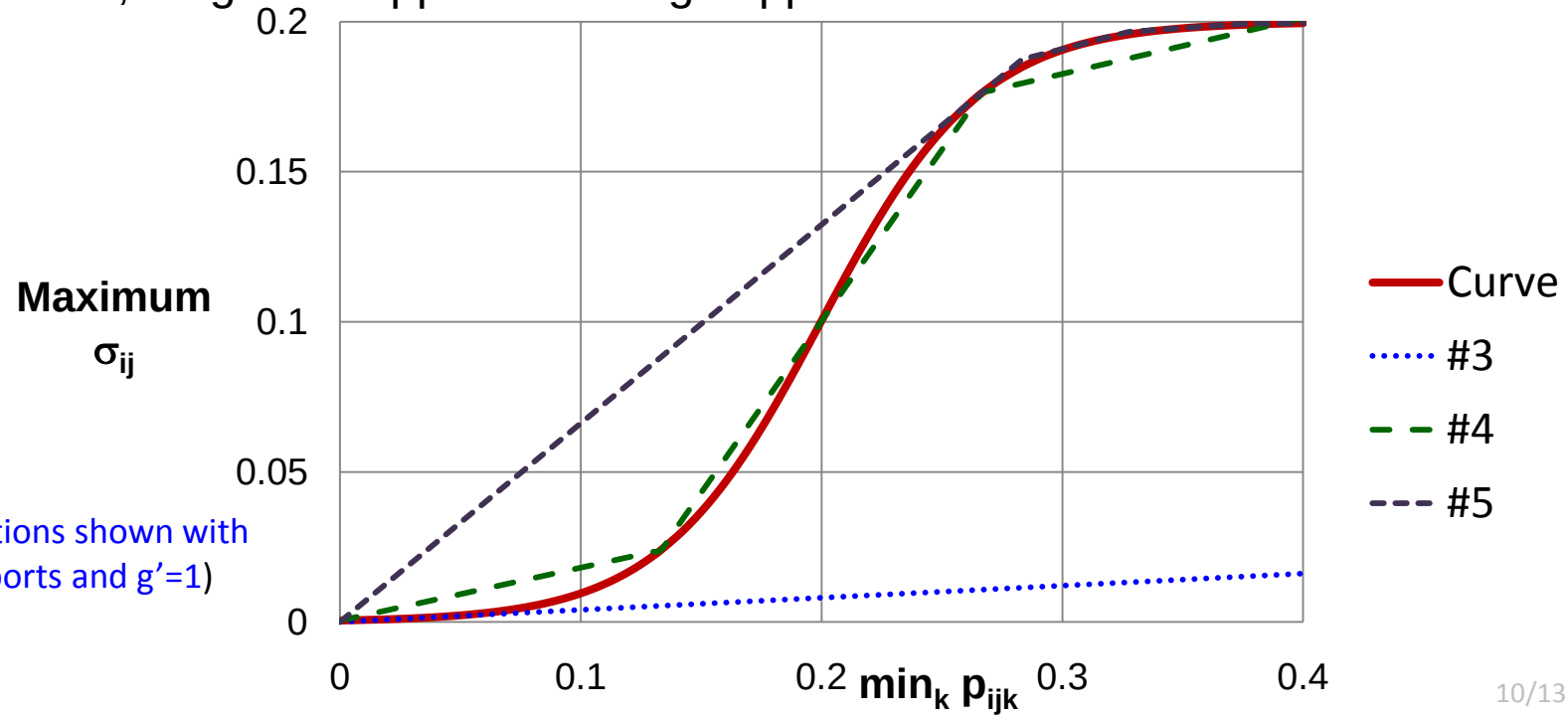
Synergy relationship used for *K*=2, *g'*=1

- Resulting MINLP formulation has nonconvex relaxation

NIP-MR with General Nonlinear Synergy: Algorithm and Heuristics Tested

<u>Algorithm</u>: Commercial software BARON

1. BARON Node 0 solution
2. Commercial software SBB
3. Inner linearization
4. Piecewise linear approximation - {2,3,4} equidistant segment breakpoints
5. Outer approximation - tangential chord over concave section and {2,3,4} equidistant, tangential upper bounding supports



(Heuristic constructions shown with 3 segments/supports and g'=1)

NIP-MR with General Nonlinear Synergy: Comparative Testing Results

- Testing conditions
  - 30 instances each on 15 problems sizes, ranging in size from a 5x5 to a 50x50 intermediary network of nodes
  - Time limit of 1800 CPU seconds
- Results
  - Reported are the average % optimality gaps from the best reported solution for each instance

Problems	Alg	H1	H2	H3	H4 (# segments)			H5 (# tngt spts)		
					2	3	4	2	3	4
1-9	0.46	0.47	0.65	0.95	1.20	1.21	1.15	0.28	0.27	0.25
1-12	–	–	0.62	0.89	1.13	1.13	1.08	0.23	0.22	0.20
1-15	–	–	499.77	0.85	1.08	1.09	1.04	0.19	0.18	0.16

1. Commercial solvers failed for large problem sizes
2. The outer linearization heuristic with 4 tangential supports was the most effective
   - Worst solution gap performance of 2.539% was superlative among techniques
   - Reduction in computational effort by a factor of 461 over BARON for Problems 1—9

  - Additional testing with H4 segments constructed to mirror H5 (1 segment over the convex region and {2,3,4} over the concave region) did not improve performance

Conclusions and Recommendations

- Results
  - Novel models that account for synergistic resource relationships
  - Effective solution methods that outperform leading commercial software SBB and/or BARON

- Future Research
  - Sociological research on the forms of superadditive synergy between groups/agencies
  - Computationally study on the effects of three or more resource types

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