Finding Optimal Strategies for Influencing Social Networks in Two Player Games

MAJ Nick Howard, USMA
Dr. Steve Kolitz, Draper Labs
Itai Ashlagi, MIT
Problem Statement

• Given constrained resources for influencing a Social Network, how best should one spend it in an adversarial system?
Agents

• Each node in the network is an agent
• 4 classes of agents: Regular, Forceful, Very Forceful, Stubborn (immutable)
• Scalar belief for each agent: $X_i(t) \in [-0.5, 0.5]$
Network and Interactions

- Arcs represent communication
- Stochastic interactions occur with one of three types:
  - Forceful w.p. $\alpha_{ij}$
    \[
    X_i(t+1) = X_i(t) \\
    X_j(t+1) = \varepsilon_{ij} \cdot X_j(t) + (1 - \varepsilon_{ij}) \cdot X_i(t)
    \]
    \[0 \leq \varepsilon_{ij} \leq 0.5\]
  - Averaging w.p. $\beta_{ij}$
    \[
    X_i(t+1) = X_j(t+1) = \frac{X_i(t) + X_j(t)}{2}
    \]
  - No Change w.p. $\gamma_{ij}$
    \[
    X_i(t+1) = X_i(t) \\
    X_j(t+1) = X_j(t)
    \]
Data Parameterization

• We choose a set of influence parameters that generally make influence flow ‘down’
• However we found through simulation that the solution to our game is highly robust to changes to these parameters.
Interaction Simulation

Attitude Scale

Pro-gov’t
Anti-TB

‘Neutral’
Fence-sitters

Anti-gov’t
Pro-TB
Interaction Simulation

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West Point
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West Point
Mean Belief of Network Over Time
Properties of the Network Model

• Well defined first moment
  – Beliefs converge in expectation at a sufficiently large time in the future to a set of equilibrium beliefs:
    \[ X^* = \lim_{k \to \infty} E[(X(t + k)|X(t))] \]
  – \( X^* \) is independent of \( X(t) \)
• In simulation the average belief of the network oscillates around the average equilibrium belief
• \( X^* \) can be calculated in \( O(n^3) \)
The Game

• Complete information, symmetric
• 2 players control a set of stubborn agents, and then make connections to the mutable agents
• Payoff functions defined as a function of the expected mean belief of network (convex combination of the elements of $X^*$)
Example Payoffs

Mean Belief: -.0139

Mean Belief: .1545
Finding Nash Equilibria

• Finding Nash Equilibria from a payoff matrix is straightforward
• However, given C connections for each player, it takes $O(n^{2C+3})$ to enumerate the payoff matrix
• Simulated Annealing runs in $O(n^3)$ to optimize a single player’s strategy (vs MINLP)
• Applying the simulated annealing algorithm allows us to use Best Response Dynamics to find Pure Nash Equilibria in $O(n^3)+$
• The equilibria turn out to be highly robust to changes of the influence parameters
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Notional Best Response Dynamics
Search Using Simulated Annealing

US Player

TB Player

Starting Payoffs

Nash Equilibria

US Player improves payoff from 1 to 3 through Simulated Annealing

TB Player improves from 2 to 3 with new solution

TB Player improves from 2 to 4

US Player improves from 1 to 4

TB Player improves from 1 to 3

US Player improves payoff from 1 to 3 through Simulated Annealing
Thoughts

- The time to find Pure Nash Equilibria increases exponentially with C
- Simulated Annealing heuristic provides a method to conduct best response dynamics
- Current equilibrium payoff functions not completely realistic, but make problem tractable
- Inverse Optimization could be used to determine the enemy’s payoff function based off of intelligence reports of villages
State: $X_k = [X_1, \ldots, X_7]$ \quad |X_k| = 3^7

$X_i \in \{-1, 0, 1\}$

Control: $u \in \{1 \text{ of the 35 Inf. Nodes}\}$

Disturbance: $P_{ij}(X_k, u)$

$$g(i) = \sum_{i=1}^{7} X_i$$

$$J(i) = \arg \max_u (g(i) + \alpha \sum_j P_{ij}(i, u)J(j))$$
Conclusions

• Even with small amount of information, I can generate a set of stationary policies that do much better than a static strategy
• Once the network goes to a large positive belief, it generally stays there (self-sustaining)
• Flexibility and information have enormous utility in affecting the flow of information in this model
Recommendations for Future Work

• Non-equilibrium payoff functions
• Different Strategy Options
• Partial information game
• Dynamic Networks