**A game-theoretic and network analysis of costly wars of attrition: dyads versus triads**

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## PART ONE OF THE PAPER: DYADS

### 1. Introduction

In the first half of this paper, we focus on those two-party (dyadic) conflicts resulting in a protracted and costly stalemate that neither side prefers, a negative-sum situation often referred to as a "war of attrition" in the game-theoretic literature (e.g., Maynard Smith, 1982). Since such conflicts are counter-productive and collectively irrational, they pose a difficult puzzle. What is the underlying logic of these conflicts; why do such conflicts occur with such regularity in the real world; and what role can game theory and network analysis play in these situations? The first half of this paper is divided into four parts. Following this introduction, part two provides some background and presents some real-world examples of costly and protracted wars of attrition. Next, part three presents an idealized two-player war-of-attrition model (sec. 4.2) as well as a more realistic $n$-player evolutionary model (4.3), while part four identifies areas for further research. The second half of this paper presents

the use of even or odd final structural components of networks to determine if it is balance or imbalanced and how that affects C2.

## 2. Background and summary of previous literature

Competitive interactions often involve conflicting claims over valuable resources (e.g., the division of licensing and television revenues among players and owners in sports leagues; disputes over land and water rights in the West Bank) or competing preferences over public policy (e.g., Puerto Rico's territorial status; the extent of the national government's role in the health care system). How are these conflicting claims or preferences to be decided or resolved?

One solution method is "spontaneous order" (Sugden, 1985), or the decentralized creation and evolution of self-enforcing norms and conventions (Lewis, 1969). Yet the resulting norms or conventions may not necessarily be wealth-maximizing or mutually-beneficial, and the conditions necessary for their creation and evolution may not be stable or present (Guerra-Pujol, 2009). Another solution method is centralized coercion (cf. Hobbes). A coercive solution, however, presupposes the existence of an external Leviathan able to impose its will, and regardless of the existence of such a Leviathan, coercive solutions are often perceived as illegitimate or unfair by the losing side, leading to further conflict and unrest (Zajac, 1995). Lastly, a more desirable method of resolving competing claims is through bargaining (Coase, 1960). In theory, negotiated solutions produce gains from trade in which all sides are made better off. In practice, however, bargaining might be unfeasible or unproductive for a wide variety of reasons, such as self-serving biases (Babcock &

Loewenstein, 1997), high transaction costs (Simester & Knez, 2002), and the existence of "infeasible" or mutually-incompatible claims (Gächter&Riedl, 2005).[1]

Consider, for example, the decades-long debate in Congress over health-care reform (Rand Corp., 2009), the disastrous Major League Baseball strike/lock-out of 1994-1995(Staudohar, 1997), and the long-standing Israeli-Palestinian conflict over West Bank settlements (New York Times, 2009b & 2009c). These types of negative-sum conflicts are not amenable to the classic solutions set forth above. As a result, when conventions, external coercion, and bargaining are not available or practicable solutions, competitive interactions often lead to a costly and protracted stalemate among the competing sides. Other real-world examples of this phenomenon include civil actions and civil litigation generally. Although a large fraction of civil actions are settled out of court (Galanter, 2004), some civil actions resemble costly and protracted war of attritions in which the litigants and their attorneys find themselves locked in a no-win situation in which neither side is willing to compromise or back down. Both plaintiff and defendant thus end up investing significant resources in the contest, but their respective efforts end up cancelling each other out, leading to a Pyrrhic victory or no victory at all for either side (e.g. Harr, 1996).

The "war of attrition" concept, often referred to as the "Hawk-Dove Game," has been extensively treated in the evolutionary biology literature (Maynard Smith & Price, 1973; Bishop & Cannings, 1978; Maynard Smith, 1982). In the first half of this paper, we present a modified war of attrition model designed to capture the essential features of the myriad conflict situations described above. Our goal is not only to explain the ultimate source of the impasse over Puerto

[1] For purposes of this background section, we have identified only three major methods of dispute resolution. Another possible conflict-resolution method, though less frequently used in practice, is a random resolution method, such as a coin toss (see, e.g., New York Times, 2009a).

Rico's constitutional status but also to understand the perverse underlying logic of costly and protracted wars of attrition generally.

## 3. A game-theoretic approach to legislative wars of attrition

In this section of the paper, we discuss the utility of mathematical models and then present a two-player dyadic war-of-attrition model as well as an $n$-player evolutionary model.

### 3.1. A few words about the utility of models

Before proceeding, we wish to make a few points about the utility of game-theoretic models. Much of the mainstream literature on the problem conflict is based on purely verbal arguments. The problem with purely verbal arguments, however, is that they tend to be imprecise, fuzzy, and vague (cf. the "level of generality" problem set forth in Tribe & Dorf, 1991, pp. 73-80). This "looseness" of most verbal arguments leads to confusion and unproductive squabbling among scholars, who end up arguing back-and-forth for years about the meaning and implications of their verbal models.

Of course, we recognize at the outset that game-theoretic models and mathematical models generally are much simpler than the real-world scenarios they are designed to depict. For example, the models in this paper are highly-stylized and based on a finite set of simplifying assumptions, such as symmetrical payoffs and a well-defined strategy set consisting of only two choices. In addition, we ignore the psychology of sunk costs, the problem of stochastic effects (such as random shocks), and the possibility of altruism or "other-regarding" behavior. Nevertheless, what models give up in specificity and detail, they gain in tractability and clarity. In the words of two contemporary game theorists: "models are simple maps for understanding the consequences

of a small number of key assumptions" (McElreath & Boyd, 2008, p. 4). In addition, formal models may help us uncover the underlying logic or unity of structure of seemingly unrelated situations. Then, and only then, after stating the operating assumptions of our models up front, do we attempt to explain the results of our models in words.

### 3.2. A two-player war-of-attrition model

Our two-player war-of-attrition model consists of a one-shot, simultaneous-move game in which the players, whom we designate as Player A and Player B, support opposing military objective or opposing positions on a given issue or public-policy debate. We begin with this simple scenario first for ease of exposition. Later in this subsection (3.2), we consider an infinite version of the game.

First, we make the following set of assumptions: each player prefers that his policy position prevail and obtains the positive payoff $v$ if his position indeed prevails.[2] Also, the players must simultaneously choose between one of two possible strategies at the outset of the game: hawk or dove. The hawk strategy is the equivalent of fighting, and there is no bluffing in this game: the player who fights always pays a cost $(-c)$ to obtain his or her desired policy goal or military objective. The dove strategy, in contrast, consists of backing down or retreating. By choosing to back down, the player does not pay the cost of fighting. Lastly, we assume that the benefit or value of prevailing $(v)$ is always larger than the cost of fighting$(c)$. That is, we assume that $v$ is greater than $c$. The intuition behind this assumption is that the players are unlikely to fight when the cost of fighting outweighs the utility of prevailing or the value of the

[2] For simplicity, we assume that the intensity of the preferences of each player are equal; that is, we assume that the value of $v$ is fixed.

contested resource, as the case may be. (As an aside, this analysis suggests that one way of preventing wars of attrition is by increasing the cost of fighting relative to the value of prevailing.)

Notice that we specify the respective benefits and costs corresponding with each strategy in abstract terms—the parameters $v$ and $c$—rather than expressing them in terms of numerical values (such as 1, 2, and 3) in order to illustrate the underlying logic and structure of seemingly unrelated problems. In addition, another advantage of expressing these values as abstract parameters is flexibility and generality; that is, our abstract model permits us to derive results for any value that these parameters might actually take (cf. McElreath & Boyd, 2008, p. 6).Since this is a game-theoretic or interactive model, the payoffs depend on the strategies simultaneously chosen by the players at the start of play. The game and the payoff-structure can be expressed in "extended form" (see Figure 1a) as follows:

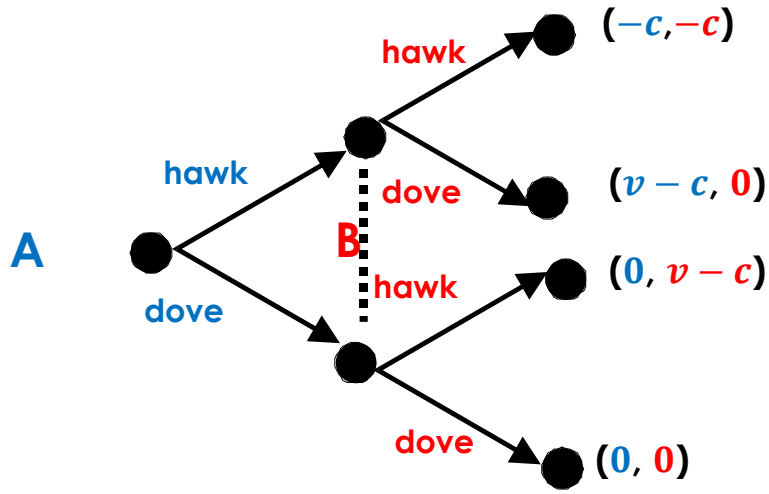


**Figure 1a**

*Extended form (game tree) symmetrical war-of-attrition game with two-players and two-*

In addition, the payoffs can also be expressed in "normal form" (see Figure 1b) as follows:

| | Player B | Player B |
|---|---|---|

| | chooses *hawk* | chooses *dove* |
|---|---|---|
| Player A chooses *hawk* | $-c, -c$ | $v-c, 0$ |
| Player A chooses *dove* | $0, v-c$ | $0, 0$ |

**Figure 1b**

*Normal form payoff table of symmetrical war-of-attritiongame with two-players and two-strategies*

Accordingly, there are four possible interactions in the one-shot game: (1) hawk-dove, (2) dove-hawk, (3) dove-dove, and (4) hawk-hawk, resulting in four possible payoff combinations. For instance, if Player A chooses hawk and Player B chooses dove, then A receives a positive payoff minus the cost of fighting ($v-c$), while B obtains a zero payoff.[3] This result occurs because Player A is more likely to obtain his desired policy change if the other player does not fight back, though A still ends up paying the cost of fighting. Likewise, if Player A chooses dove while Player B plays hawk, then the payoffs are reversed: A gets nothing and B gets $v-c$.

In the event of a dove-dove interaction in which both players choose to back down instead of fighting, neither prevails or obtains his preferred policy outcome, and thus both players receive a zero payoff. The reason for a zero payoff (instead of $v/2$ as in the classic hawk-dove game) is that in our model both players prefer change to the status quo, but when both players back

[3] The payoff to the dove player in this scenario might, in fact, be negative. For instance, as my research assistant Nadine Sfeir has pointed out, although such a player avoids the cost of fighting, his choice of the dove strategy might generate a reputation for backing down or might weaken his ability to fight in future rounds. Nevertheless, since our model consists of a single, one-round game, we ignore reputation costs and the like for simplicity.

down, the status quo is preserved by default.[4] Lastly, if both players choose to fight, resulting in a hawk-hawk interaction, then both players end up paying the cost of fighting ($-c$) and neither prevails because, by fighting, their efforts are likely to cancel each other out, leading to a preservation of the status quo, a state of affairs that neither player prefers.

At this point, it is worth noting that one aspect of our war-of-attrition model shares some similarities with a model often referred to as the "Game of Chicken" (e.g.Poundstone, 1993, pp. 197-201). In the Game of Chicken, there are two players driving race cars towards each other at high speeds on a one lane road. Like the binary strategy set in our model, the strategy set of the drivers in the Game of Chicken consists of two choices: swerve or drive straight. If both players decide to drive straight, there will be a serious collision and thus a cost will be imposed on both drivers. Likewise, if the players in our model both decide to fight, both players end up paying the cost of fighting.

Nevertheless, our model differs from the Game of Chicken in two important dimensions. One difference is the treatment of cooperative interactions, such as "swerve-swerve" and "dove-dove" interactions. In our model, dove-dove interactions produce a zero payoff for both players since both avoid the cost of fighting and since neither is able to impose his desired outcome on the other player. In the Game of Chicken, however, swerve-swerve interactions produce a small positive payoff for both players because both drivers avoid being the sole "chicken."

[4] But as my colleague Carlos del Valle has pointed out, a dove-dove interaction might produce a positive payoff, a "peace dividend." Here, however, we postulate a zero payoff since neither player prefers the status quo. Nevertheless, we will relax this assumption in a subsequent paper by presenting a model in which one of the players prefers the status quo to change.

Another difference between our model and the Game of Chicken is the treatment of mixed-type interactions, such as "swerve-drive straight" and "dove-hawk" interactions. In our model, when a dove interacts with a hawk, the dove's payoff is zero. He avoids the cost of fighting, and the other player's win does not impose a cost on the dove. In contrast, the worst possible outcome in the Game of Chicken is to swerve if the player is going to drive straight. The player that swerves not only obtains a negative payoff (since he has lost the contest), but also this negative payoff is greater than the negative payoff generated when both players drive straight. The logic of this result is that the driver that swerves is the sole "chicken." That is, although a head-on collision also produces a negative payoff, neither player can be called a "chicken," and by definition, being called this epithet is deemed worse than the cost of a head-on collision.[5]

Before proceeding, it is also worth taking a moment to describe the salient differences between the hawk-dove model presented in this paper and the traditional or classical hawk-dove model that appears in the game theory literature (e.g. Maynard Smith & Price, 1973; Maynard Smith, 1982).[6] In brief, our modified war-of-attrition model differs from the Maynard Smith model in the following respects:

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[5]The Game of Chicken thus raises an intriguing possibility: under what conditions might the dove strategy in our model generate a negative payoff? For example, there might be a cost to backing down (choosing dove) when the other player chooses hawk, such as a reputation cost. In addition, backing down at time T1 might affect the outcome of a future contest at time T2. That is, one's choice of strategy may have a "path-dependent effect." For example, in the standard war-of-attritions model in the existing literature (see, e.g., Maynard Smith, 1982), the outcome of hawk-hawk and dove-dove interactions are completely random. But one could imagine a more complex or dynamic model in which the choice of strategy during one round of play increases or decreases the distribution of payoffs in future rounds of the game. The intuition behind this idea is as follows: if I play hawk and you dove during the first round of play, I not only win the first round, I am also in a better position to win future rounds of play even if you later switch to hawk. Nevertheless, we ignore this possibility in our model for the sake of simplicity.

[6] The first paper to present the hawk-dove model was Maynard Smith and Price (1973). Maynard Smith (1982) presents the hawk-dove model in its traditional or canonical form, which we shall refer to as the "Maynard Smith model" for reference, in contrast to our model, which we shall refer to as the modified hawk-dove model.

(a) biological versus social science approach

The focus of the classical Maynard Smith model is animal conflict. Specifically, the Maynard Smith model is designed to explain the prevalence of "display" behavior in nature in which many species of animals settle contests without resort to actual fighting. The focus of our model, in contrast, is on actual human conflict, such as conflicts over public policy or scarce resources, and not on mere display behavior. (In section 4.3 of this paper below, though, we do develop an evolutionary or biological war-of-attrition model.)

(b) dove-dove interactions

Dove-dove interactions in the Maynard Smith model produce a payoff of $v/2$ for both players because the players are assumed to share the resource or, in the alternative, because the winner of the contest is selected at random, so that each player wins half of the time. In our model, by contrast, we assume more realistically that when both players choose to back down, neither player wins the contest, leading to a zero payoff (instead of $v/2$) for both players.

(c) hawk-hawk interactions

Lastly, in the Maynard Smith model the outcome of a hawk-hawk interaction is determined at random, leading to a payoff of $(v-c)/2$, because the players are assumed to be equally-matched. Our model likewise assumes that the players are equally matched, but we reach a fundamentally different conclusion from this baseline assumption. We conclude that neither player wins the contest, leading to a negative payoff of $-c$ instead of $(v-c)/2$ for both players, since their efforts are likely to cancel each other out, producing a costly war of attrition and preserving the status quo.

This approach to hawk-hawk interactions provides the key to understanding the inherently negative-sum nature of costly and protracted wars of attrition: when both sides choose to fight, neither side wins because a hawk-hawk interaction increases the likelihood of stalemate. Why? Because when both sides choose to expend resources fighting for their respective policy outcomes, their efforts are likely to cancel out, leading to a costly war of attrition with no change in the status quo. Consider once again the Puerto Rican status debate and the other examples set forth in section two of this paper: the competing health-care reform proposals in Congress, the Major League Baseball strike of 1994-1995, and the Israeli-Palestinian conflict over West Bank settlements. Both sides in these myriad conflicts prefer to see their respective positions prevail, but by refusing to negotiate or surrender to the demands of other side, the warring factions end up perpetuating a negative-sum conflict in which all sides are worse off.[7]

Nevertheless, despite these differences, our modified model shares the same underlying logic and structure of the Maynard Smith model. In both models, the individual players are competing against each other, they are acting as proxies for strategies (either hawk or dove), and their strategies are set at the start of the game. Also, in both models the hawk strategy (fighting) can be compared to defection, while dove (backing down) is more akin to cooperation. That is, if both players choose to back down, then neither gets its preferred outcome, but neither pays the cost of fighting either, perhaps allowing time for a mutually-beneficial compromise solution or a negotiated settlement. If both choose to

[7] For example, the existence of military checkpoints and barrier walls between Israel and the West Bank imposes a significant cost on both Palestinians and Israelis. Similarly, neither the players nor the owners in the Major League Baseball strike benefitted from the cancellation of 1994 World Series. Likewise, Puerto Rico's main political parties have expended considerable resources promoting their preferred status options, but thus far their efforts appear to have cancelled each other out, ultimately resulting in no change in the Puerto Rican status quo.

fight, however, neither obtains its preferred outcome, yet both end up paying the cost of fighting.

Having compared and contrasted the classical Maynard Smith hawk-dove model with our modified war-of-attrition model, we now proceed to find the existence of Nash Equilibria in the modified model. That is, what is the likely outcome of our revised hawk-dove game?

In summary, there are two pure-strategy equilibria in the modified one-shot game. This conclusion becomes apparent once we evaluate each player's best response to the other. Given the payoff structure of our modified hawk-dove model (as set forth in Figures 1a and 1b above), hawk is the best response to dove because is $v - c$ is greater than $0$, while dove is the best response to hawk because $0$ is greater than $-c$ . That is, Player A prefers to fight if Player B is going to back down, and vice-versa, B prefers to back down if A is going to fight. Nevertheless, this is not the outcome we observe in the war-of-attrition scenarios described in sections two and three above. In the absence of conventions, external coercion, or bargaining, we saw that a conflict might often escalate into a costly and protracted war of attrition. How can we explain the persistence of such wars of attrition, i.e. hawk-hawk interactions, in so many myriad types of conflict scenarios?

One source of this incongruity is the simultaneous-move nature of the modified hawk-dove game. Being a simultaneous-move game, neither player can observe the other player's move ahead of time, so the players could end up choosing the same strategies (hawk-hawk or dove-dove), leading to sub-optimal payoffs for both players. For example, both players might be committed ex ante to fighting—hoping in vain that the other player has chosen to back down—leading to a costly and pointless war of attrition in which both players end up fighting.

Another source of negative-sum hawk-hawk interactions is the possibility of mixed strategies. In place of a single, pure strategy (i.e. hawk or dove), the players might select a probabilistic mix of hawk-dove combinations, playing the hawk strategy, for example, with probability $p$ and choosing dove with probability $1-p$. (In baseball, for example, the pitcher will mix his repertoire of pitches—e.g. fastball, curve ball, sinker, slider, etc.—in some probabilistic or random manner.) Once we introduce the possibility of mixed strategies, we can easily determine the probability of a hawk-hawk interaction, that is, the probability that the game will turn into a war of attrition.

Consider an infinite version of our two-player hawk-dove war of attrition game. That is, instead of a one-shot or single-round game, we now imagine a potentially never-ending game consisting of an infinite or endless number of rounds.[8] In summary, the payoff structure of the infinite game is identical to that of the one-shot game (cf. Figures 1a and 1b), since each individual round of the infinite game provides the same payoffs to the players as the one-shot game does. Furthermore, we assume the players avoid the "sunk cost fallacy"; in other words, when playing a given round of the infinite game, each player ignores his previous payoffs (i.e., his accumulated losses and wins during the previous rounds of play) because such previous payoffs are, in essence, sunk costs and thus have no bearing on the current round being played.[9]

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[8] For example, returning to some of the conflict scenarios set forth in section two above, we could model the long-standing debates over Puerto Rican status and health-care reform or the Mideast conflict as an endless hawk-dove game. Indeed, this idea is explicit in H.R. 2499, the Puerto Rican status bill currently pending before Congress, which proposes a potentially never-ending series of referenda if a majority of the voters fail to approve Option 2, the change option.

[9] The fallacy consists of the assumption that a player's prior investment in fighting costs commits him to future investment. Instead, each player should consider his expected or future payoffs in deciding his optimal strategy and thus in deciding whether to continue playing at all (see, e.g., Dawkins, 1989, p.150). Of course, in ignoring sunk costs, we are assuming that the players are acting rationally in the *homo economicus* sense. Later, in section 4.3 below, we relax this rationality assumption.

(Before proceeding, notice that the infinite version of the game will end only when one or both of the players decide to stop playing the game, either because one of the players has accumulated too many losses during the course of the game, leading to his voluntary surrender, or because both players are able to opt-out of the game altogether through a negotiated settlement or compromise solution. In our model, however, we assume that the player have an unlimited amount of resources to fight and that negotiations are not feasible.)

What is the optimal strategy in the infinite game? Since neither player knows with certainty the strategy of the other player, the optimal approach might consist of a mixed strategy in which a player is indifferent to playing either hawk or dove.

Consider Player A first. (In fact, the analysis in the remainder of this section applies equally to both A and B since the payoffs in this model are symmetrical for both players.) Here, we introduce the related ideas of expected payoff and probability. That is, Player A's expected payoff from playing a given strategy (say, hawk) depends on the probability ($p$) that Player B might also play hawk as well as the probability ($1-p$) that B might choose dove.

Thus, if Player A chooses hawk, A will pay a cost $-c$ with probability $p$ and will obtain the payoff $v-c$ with probability $1-p$. (Recall that $p$ is the probability that Player B also plays hawk, while $1-p$ is the probability that B plays dove.) Accordingly, the expected payoff ($V$) to Player A of playing hawk $[V(H)]$ can thus be expressed mathematically as follows:

$$V(H) = (-c)(p) + (v-c)(1-p) \tag{1.1}$$

Likewise, Player A's expected payoff from choosing the dove strategy $[V(D)]$ can be expressed in the following form:

$$V(D) = (0)(p) + (0)(1-p) = 0 \tag{1.2}$$

That is, if Player A chooses dove, then A receives a zero payoff because, regardless of the other player's strategy, the dove strategy always produces a zero payoff against both hawk and dove, given our initial assumptions.

Lastly, we determine Player A's optimal mixed strategy$[V'(A)]$by setting equations 1.1 and 1.2 equal to each other, substituting $p'$ for $p$, and solving for $p'$as follows:

$$(-c)(p') + (v-c)(1-p') = 0$$

$$-cp' + v - c - vp' + cp' = 0$$

$$v - c - vp' = 0$$

$$vp' = v - c$$

$$p' = (v-c)/v = V'(A) = V'(B) \tag{1.3}$$

In other words, Player A's optimal mixed strategy is to play hawk with a probability equal to $(v-c)/v$ and dove with probability $1-(v-c)/v$. Also, notice that the optimal mixed strategy is actually the same for both players—that is, $V'(A)$ is equal to $V'(B)$—since the payoffs in this modified model are symmetrical.[10]

Summing up, then, assuming $v$ is greater than $c$, each player's optimal strategy in the infinite game is to play hawk (fight) with a probability equal to the ratio that appears on the right-hand side of the above equation. This ratio is a function of the stakes of the contest ($v$) and the cost of fighting ($c$) and

[10] Also, we present the complete details of our algebraic analysis so that the careful reader can follow each step of our analysis.

increases as the numerical value of $v$ increases and decreases as the value of $c$ increases. Thus, when $v > c$, there is a proportional relationship between the variables $p'$ and $v$ but an inverse relation between $p'$ and $c$.

Having determined the probability that either player will player hawk (cf. equation 1.3 above), we now find the probability that any given round of play will result in a hawk-hawk interaction, that is, the probability that both players will simultaneously play hawk on round $n$ of play, where $n$ is any individual round of play. In summary, since both players will choose the hawk strategy with probability $p'$ due to the symmetrical payoff structure of the game, the probability that both players play hawk on any given round of play is simply $p' \times p'$, or $p'^2$. (Similarly, the probability that hawk-hawk interactions occur on two consecutive rounds of play is $p'^2 \times p'^2$, or $p'^4$.)

Since the value of $p'$ (and thus the value of $p'^2$) is a function of the stakes of the contest ($v$) and the cost of fighting ($c$), our modified model conveys a valuable insight about real-world symmetrical wars of attrition. In essence, the lesson is this: the larger the stakes of the conflict are (i.e. the higher $v$ is), the larger the probability of fighting. By the same token, the larger the cost of fighting is (the higher $c$ is), the smaller the probability of fighting.

Thus far, we have presented a two-player war of attrition of model. In the next section, we consider a more realistic $n$-player evolutionary model.

### 4.3.An $n$-player evolutionary war-of-attrition model

In place of the two-player model above, we now consider an $n$- or multi-player hawk-dove model. Our $n$-player model operates on the following assumptions: the population consists of a large number of individuals; during each round of play, two individuals selected at random from the population meet and engage

in a micro-conflict; and as before, there are only two possible strategies or types of individuals: hawk and dove. Lastly, we further assume that these strategies are transmitted through an asexual inheritance mechanism: the victor of each two-pair, micro-conflict not only survives but also produces a descendant-clone who asexually inherits the victor's strategy; the loser, in contrast, is eliminated from the population. If the conflict ends in a draw or tie, both contestants survive but neither produces a descendant.

Given these assumptions, we ask which strategy will be favored by natural selection? To answer this question, we proceed in stages. First, we restate the payoff $(V)$ corresponding to each possible interaction. Since the payoffs are the same as before (cf. Figure 1b), and since there are four possible micro-interactions in all (hawk-hawk, hawk-dove, dove-hawk, and dove-dove), the payoffs corresponding to each interaction can be written as follows:

$V(H \mid H) = -c$ (i.e. the payoff to a hawk, given that it interacts with another hawk)

$V(H \mid D) = v - c$ (the payoff to a hawk, given that it interacts with a dove)

$V(D \mid H) = 0$ (the payoff to a dove, given that it interacts with a hawk)

$V(D \mid D) = 0$ (the payoff to a dove, given that it interacts with another dove)

On the left-hand side, we have written the payoffs corresponding to each interaction in mathematical form, and on the right-hand side, we have "translated" the mathematical notation into plain English for the non-

mathematical reader. Notice too that, because of the symmetrical nature of our model, for simplicity we state the payoffs of the row player only (Player A).

In a population model, the fitness of a given strategy is said to be "frequency dependent" because the success or fitness of a strategy depends not only on the frequency of the other strategy but also on that strategy's own frequency (cf. McElreath & Boyd, 2007, p. 38). When fitness (rate of survival) is frequency dependent, we proceed to use evolutionary game theory to determine which strategies are "evolutionarily stable strategies" or ESSs and to find the long-run evolutionary equilibrium of the population—that is, the frequency of hawks and doves over many generations. Specifically, we wish to answer the following two questions:

(a) is either strategy able to resist invasion by the other strategy?

(b) if no strategy is evolutionarily stable (i.e. able to resist invasion by the other strategy), is there an evolutionarily stable mix of strategies?

Let $p$ be the frequency of hawks in the population and $1-p$ the frequency of doves in the population. First, consider a population in which the frequency of hawks is very high $(p \approx 1)$. Under this scenario, hawks rarely interact with doves because the frequency of doves is very low $(1-p \approx 0)$, and thus the average fitness of a hawk $[W(H)]$ is determined by his interactions with other hawks as follows:

$$W(H) = w' + (1)[V(H|H)] + (1-1)[V(H|D)]$$

$$W(H) = w' + V(H|H)$$

$$W(H) = w' - c$$

Before proceeding, note that the parameter $w'$ in our equations always refers to the "baseline fitness" of all the individuals in the population—that is, the

probability of survival from generation to generation—and thus reflects the strength of selection on a given population (cf. McElreath & Boyd, 2007, pp. 40-41). For convenience, we assume that $w'$ is larger than $c$, the cost of fighting.

Now, consider the possibility of a rare dove-like mutant. What would happen if a hawk were to interact with this rare dove mutant? Will the dove strategy begin to spread across the population, gradually displacing the hawks, or will the hawk strategy be able to resist invasion? To answer this question, we determine the average fitness of the rare dove mutants among the population of hawks. Since doves are rare $(1 - p \approx 0)$, the chance one dove will meet another dove is likewise small. As a result, the average fitness of a rare dove $[W(D)]$ is determined by his interaction with hawks as follows:

$$W(D) = w' + V(D|H)$$

$$W(D) = w' + 0 = w'$$

Notice, then, that in a population of hawks, the rare doves will on average outperform the hawks because $W(D)$ is larger than $W(H)$. In other words, doves will invade the population and displace hawks because the average fitness of each dove $(w')$ is on average higher than the average fitness of each hawk $(w' - c)$. Hawks will not be able to resist an invasion of doves.

But can a population of doves resist an invasion of hawks?

Consider a population in which the frequency of doves is very high $(1 - p \approx 1)$. Since doves rarely interact with hawks when the frequency of hawks is low $(p \approx 0)$, the average fitness of a dove $[W(D)]$is determined by his interactions with other doves as follows:

$$W(H) = w' + (1)[V(D|D)] + (1 - 1)[V(D|H)]$$

$$W(D) = w' + V(D|D)$$

$$W(D) = w' - 0 = w'$$

What would happen if a rare hawk-like mutant were to appear on the scene? Since hawks are rare $(p \approx 0)$, the hawk mutant will interact mostly with the doves and his average fitness $[W(H)]$ is thus as follows:

$$W(D) = w' + V(H|D)$$

$$W(D) = w' + v - c$$

Thus, in a population of doves, the rare hawks will on average do better than the doves because $W(H)$ is larger than $W(D)$ when the baseline fitness is larger than the cost of fighting. Just as doves will invade a population of hawks, in this case hawks will invade a population of doves and eventually displace the doves because the average fitness of each hawk $(w' + v - c)$ is on average higher than the average fitness of each dove $(w')$. Accordingly, these results suggest that neither hawk nor dove is an evolutionarily stable strategy (ESS) because neither strategy is able to resist invasion by the other. But is there is some stable mix of hawks and doves? And if so, what is it? That is, in the long-run, what proportion of the population will be hawks, and what proportion will be doves?

To find this mixed or "polymorphic" population equilibrium, we must first determine the expected or average payoff $(W)$ corresponding to each strategy across a large number of interactions. Since the expected payoffs depend on the overall frequency or proportion of hawks and doves in the population, let $p$ be the frequency of hawks in the population and $1 - p$ the frequency of doves in the population. Then, assuming that individuals interact at random, we obtain the expected fitness of a hawk as follows:

$$W(H) = w' + (p)\left[V\left(H \,\middle|\, H\right)\right] + (1 - p)\left[V\left(H \,\middle|\, D\right)\right]$$

$$W(H) = w' + (p)(-c) + (1-p)(v-c)$$

$$W(H) = w' - pc + v - c - pv + pc$$

$$W(H) = w' - pc + v - c - pv \qquad (2.1)$$

Similarly, we obtain the expected fitness of a dove as follows:

$$W(D) = w' + (p)\left[V\left(D \mid H\right)\right] + (1-p)\left[V\left(D \mid D\right)\right]$$

$$W(D) = w' + (p)(-0) + (1-p)(0)$$

$$W(D) = w' + 0 + 0$$

$$W(D) = w' \qquad (2.2)$$

(Again, recall that the parameter $w'$ in both equations 2.1 and 2.2 is the average fitness or baseline fitness of individuals in the population.)

Having determined the expected fitness of hawks and doves, we now set $p = p'$ and $W(D) = W(H)$ and solve for $p'$ to find the frequency at which there is an evolutionarily stable mix of hawks and doves in the population:

$$W(D) = W(H)$$

$$w' = w' + v - c - p'v$$

$$p'v = v - c$$

$$p' = (v-c)/v$$

(2.3)

Notice the striking parallel between the results of our evolutionary population model and results of the standard two-player game-theoretic model in the previous subsection of this paper. Specifically, under the standard model an individual player's optimal mixed strategy is to play hawk with a probability equal to$(v-c)/v$ (cf.equation 1.3 in subsection 4.2 above). Similarly, the equivalent evolutionary modeldemonstrates that the evolutionarily stable

proportion of hawks in a large population is also equal to the same ratio: $(v - c)/v$.

In addition, like our previous model, the evolutionary model produces two qualitatively different outcomes depending on the ratio of $v$ to $c$. When the value of the resource exceeds the cost of fighting $(v > c)$, hawks will displace doves from the population, and the larger $v$ is in relation to $c$, the greater the proportion of the population will consist of hawks. But when the cost of fighting exceeds the value of the contested resource $(v < c)$, then doves will outperform hawks, and the larger $c$ is in relation to $v$, the greater the proportion of doves will be. In both cases, the population will consist of a mix of doves and hawks, with the actual proportion of this mix depending on the magnitudes of $v$ and $c$.

## 4. Discussion

Competing interactions over scarce resources or over the content of public policy can result in radically different outcomes. Consistent with Adam Smith's "invisible hand" metaphor of markets, some interactions might lead to a positive-sum equilibrium in which everyone benefits from mutual cooperation (e.g. Trivers, 1971). Other interactions, in contrast, might produce a negative-sum Hobbesian equilibrium in which everyone is worse off because of mutual defection (e.g. Poundstone, 1993). For example, when conventions, coercion, and bargaining are not available for achieving a positive-sum outcome, a competitive interaction might result in a costly and protracted war of attrition. Furthermore, negative-sum wars of attrition abound in the real world. Consider, once again, the centennial debate over Puerto Rico's constitutional status, the impasse in Congress over health-care reform, the stand-off between the player's union and baseball club owners during the Major League Baseball strike/lock-out of 1994-1995, and the long-standing conflict between Israel and Palestine

over disputed water rights and the construction of Jewish settlements in the West Bank. In all these conflicts, the contending parties are locked in a costly and protracted negative-sum war of attrition.

In this paper, we have attempted to determine the conditions under which a hawk-hawk interaction or costly war of attrition is most likely to occur, and we have shown that when the payoffs are symmetrical and bluffing is not permitted, the probability of a player choosing the hawk strategy (or the proportion of the population consisting of hawks) is a function of the stakes of the contest ($v$) and the cost of fighting ($c$) and increases as the numerical value of $v$ increases and decreases as the value of $c$ increases. In summary, the larger the stakes of the conflict are (i.e. the higher $v$ is in relation to $c$), the larger the probability of fighting (or the greater the proportion of hawks in the population). In the alternative, the larger the cost of fighting is (the higher $c$ is in relation to $v$), the smaller the probability of fighting (or the smaller the proportion of hawks in the population).

In addition, we wish to conclude this paper by identifying the following areas for future research:

(a) what happens when the payoffs are asymmetrical instead of symmetrical?

In this paper paper, we have assumed for the sake of simplicity that the cost of fighting ($c$) and the stakes of the contest ($v$) are symmetrical for the players. In reality, however, although both players might prefer some form of change to the status quo, they will oftentimes disagree on the extent of the desired change. One player might prefer a radical or “hard” change to the status quo (the “hard player”), while the other player might prefer a small or “soft” change (the “soft player”). In other words, the stakes of the conflict and thus the cost of fighting might be asymmetrical for the players.

For example, returning to H.R. 2499, the debate over Puerto Rico's constitutional status is, in essence, a strategic contest between hard and soft players in which the existing Commonwealth status serves as the status quo position. The NPP and the PIP support the statehood and independence options, respectively. Both options constitute hard changes to the status quo. The PDP, in contrast, prefers an "enhanced" or improved Commonwealth, a softer or more incremental change. But many members of the PDP as well as the PIP prefer the status quo to statehood; that is, they perceive the Commonwealth status as a "lesser evil" than statehood.

(b) what happens if the values of $v$ and $c$ change over time?

Another interesting question for future research is to consider the possibility that $v$ and $c$ might not be fixed but might change over time. That is, the payoffs might be dynamic instead of static. For example, the value of the contested resource might grow with time. By the same token, the cost of fighting might vary over time as tactics and technology change. Thus, we could build a model in which either $v$ or $c$ (or both) change by some constant $k$. This constant might be fixed in which $v$ increases by some factor $k'$ during each round of play, or it might be probabilistic, varying from 0 to 1 according to some random mechanism.

(c) is there a link between the payoffs in a given round and the outcome of future rounds?

Lastly, another interesting idea for future work is the possibility of the players becoming stronger or weaker over time, i.e. during successive rounds of play. Consider many popular video games, in which the strength of the life of the player is presented visually in the form of a bar at the top of the video screen or computer monitor. As the bar becomes smaller, the life of the player is

more at risk, and conversely, as the bar grows larger, the player's life is stronger. Similarly, one could imagine a model in which previous gains increase the probability of future gains—and in which losses during previous rounds of play increase the probability of future losses. The intuition here is that one's previous choices will not only have an effect on one's future choices (cf. "path dependence"), but will also affect the likely outcome of the future rounds of the game. For example, instead of the outcome of a hawk-hawk interaction or dove-dove interaction being decided randomly, as in the Maynard-Smith model, the outcome of such interactions might be based on the relative strength of each player, which in turn would depend on the outcome of previous rounds of play.

Also, in this paper we do not consider the accumulated payoffs (both losses and gains) from previous rounds of play, since such payoffs are, in effect, sunk costs. But in real life, the outcomes of previous rounds of play do have an effect on the psychology of the players. Moreover, once we assume that the selection and implementation of a strategy is not costless (cf. Coase, 1937) and that the resources available to the players are scarce or subject to some upper limit, then it becomes imperative to keep track of losses and gains during the rounds of play.

One possibility for doing this might be to present the model in visual form. For example, whether the players are contesting rights to land, or to access to water, or to shares of revenue, the sum value of the contested resource (e.g., land, water, revenue) might be presented visually in the form of a circle. This circle, in turn, would consist of two colors, such a blue and green, with blue representing one player's share of the contested resource and green representing the other player's share. The idea here is to present visually the

changes over time to each player's share of the contested resource during successive rounds of play.

## PART TWO OF THE PAPER: TRIADS, EVEN AND ODD NETWORKS

### I. INTRODUCTION

The period-doubling and chaos diagrams developed in the field of Complexity show stages of bifurcations that occur in natural systems and that consist of a steady state, period one, two, four and chaos. [11] The period-doubling and

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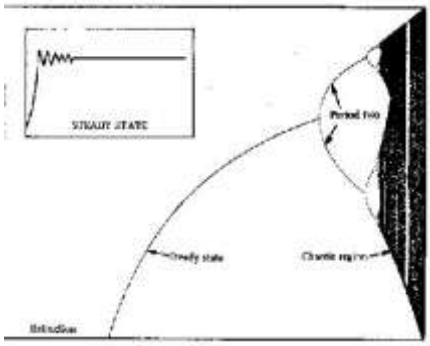
[11] See: JAMES GLEICK, CHAOS, MAKING A NEW SCIENCE, 69-80 (1987) and TRINH XUAN THUAN, CHAOS AND HARMONY, PERSPECTIVES ON SCIENTIFIC REVOLUTIONS OF THE TWENTIETH CENTURY 116-117 (2006). In both of these books they present diagrams of the different periods from an equilibrium stage up to a chaotic one. A diagram similar to the one presented is the following followed by the description made by James Gleick, Id. at 71:




> Period-Doublings and Chaos. Instead of using individual diagram to show the behavior of populations with different degrees of fertility, Robert May and other scientist used a "bifurcation diagram" to assemble all the information into a single picture.
> The diagram shows how changes in one parameter-in this case, a wildlife populations' "boom-and-bustiness"-would change the ultimate behavior of this simple system. Values of the parameter are represented from left to right; the final population is plotted on the vertical axis. In a sense, turning up the parameter value means driving a system harder, increasing its nonlinearity.
> Where the parameter is low (left), the population becomes extinct. As the parameter rises (center), so does the equilibrium level of the population. Then, as the parameter rises further, the equilibrium splits in two, just as turning up the heat in a convecting fluid causes an instability to set in; the population begins to alternate between two different levels. The splitting, or bifurcations, come faster and faster. Then the system turns chaotic (right), and the population visits infinitely many different values.

I personally have observe bifurcation phases in which a change of symmetry occurs in front of my house with a big palm tree and in the back patio with an orange tree I had. When looking at a tree imagine its initial stages starting with the planting of the seed. I did just that when I was looking at my former orange tree and then applied the Period-Doubling diagram I had just read about and theorized that the seed or the trunk was the steady state that after some time and exposure to the soil, water and sun reaches a threshold level upon which it develops a period two steady state equilibrium represented in the trunk and/or branches and continues its path until it reaches a fourth threshold level bifurcation stage upon which we see the trunk extend into branches or the branches including leaves.

chaos diagram show a natural pattern that should be studied deeply in C2 time and spatial settings. Here we propose that the evolution of the stages follows an even and odd structural characteristic that helps us assess balance and imbalance in networks.

"*The life of the law has not been logic; it has been experience*" Oliver Wendell Holmes, Jr.[12] Inspired in these words that continue to have relevance to some in the legal profession and in our way of life is taken to another epistemological level. Experience will take a leading role in the conservative network analysis approach herein developed relying on our three dimensional experience with nature, dimensions our evolutionary history has sharpened to guide us in our relation with other finite bounded elements of space that share infinite self-similar "scale-invariant"[13] characteristics. These "scale-invariant" features we best perceive in our three dimensional World will let us better understand, defend and confront the stresses and challenges natural dynamics present.

The development of network focuses on how a single actor begins relations to others forming a network of which focus is given to single vertex, two vertices (dyad) and three vertices (triad) with their respective dynamics. An illustration of why one actor and two actor relations, as depicted in networks is easier to measure, visualize and understand than three actor relations that form a triangular Euclidean geometric figure is also presented. The triangle will be

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I then thought of the chaos random phase, what was it? It then struck me, the seeds. In the case of the orange tree, the seeds of the orange fruit would be the system reaching chaos since it's probability of getting planted in fertile soil is very random and chaotic. The seed in one form or another shall end up in the terrestrial cortex but when, where and how is very complex.

[12] OLIVER WENDELL HOLMES, JR., THE COMMON LAW 1 (Am. Bar Ass'n, 1963(1881).

[13] Gleick supra note 3 at 83 et. seq.(chapter on a Geometry of Nature explaining the contribution of Benoit Mandelbrot, father of fractal theory which includes these essential self-similarity quality of fractals.)

correlated with our third dimensional form from which more complex levels of relations can be analyzed.

The exploration of these odd triadic geometrical forms will be further enriched by exploring the challenges, frustrations and defeats that triads through the "Three-body problem" presented to the epistemology of natural phenomena. This will take us into the issue that defeated the main scholars Johannes Kepler and Isaac Newton after solving the "Two body-Problem", a problem that Newton gave up on trying to tackle and that also defeated subsequent mathematical geniuses that tried to tackle it, such as the father of Graph Theory, Leonhard Euler (1707-1783), Joseph-Louis Lagrange (1736-1813) and Pierre-Simon de Laplace (1749-1827). The interest to tackle this issue will lead us to venture on how Jules-Henri Poincare (1854-1912), while working on the "three-body problem" became a pioneer of Chaos theory when revealing that indeterminacy and chaos reside at the very heart of Newton's deterministic equations, equations that worked well for single and two body networks but that break down when three bodies are involved.[14] The nature of the problem identified by Poincare I have recently coined as RUI parameters because they share the inherent limitations any model of natural phenomena will have of being relative, uncertain and incomplete pursuant to the contributions of Albert Einstein (1879-1955) with the Theory of Relativity, Werner Heisenberg (1901-1976) with the "uncertainty principle" and Kurt Gödel (1906-1978) "incompleteness theorem."[15] Hence, based on the experience of how systems evolve from one, two and three vertex relations we move to how "network

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[14] TRINH XUAN THUAN, CHAOS AND HARMONY, PERSPECTIVES ON SCIENTIFIC REVOLUTIONS OF THE TWENTIETH CENTURY 73-75, 82, 347(2006)(Describing the history with details of the actors and issues regarding the three-body problem)

[15] Orlando I. Martinez-Garcia, *The Person in The Law, The Number in Math: Improved Analysis of the Subject as Foundation for a Nouveau Regime*, Vol. 18, Number 3, American Journal of Gender, Social Policy & the Law, 507-508 (2010)

science" adopted the theories of George Simmel, considered the father of modern sociology, who developed the idea that triads are the "holy grail" of sociological and economical analysis in our complex systems. Nevertheless, NetSpids[16] working with triads seem to ignore or have at the margin the experience of other scholars with the "three body problem." Hence, this paper has the hope to mitigate waste of talent and resources in the analysis of triads in network science by presenting in general terms the experience of mathematicians with the three body problem while expanding on refined epistemological theory of knowledge based on our three dimensional classical perception vindicated with the advancement of fractal theory.

With this exposure we will then have enough elements to have a bird's eye look at why "network science" has been path dependent and hence locked in to its past inefficiently and how we can improve "network science" method using what our history and experience has perceived in a three dimensional World that has worked efficiently for us in a relativistic Universe. We propose two models, of which one is the Orlando-Manatí Model ("OM Model") to be developed in this paper as a source of C2.

In "Network Science" angles are rarely considered when measuring and visualizing the structure and operation of vertices/actors and edges/relations.[17]

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[16] In thinking on how to refer to network analysis scholars for this sentence, besides network analyst I thought on networks and their most antique and specialized creators in our environment. Vladimir Batagelj and Andrej Mrvar have identified that actor to be the spider and as such they named their large network analysis software Pajek, Slovene language word for spider. Based on this history I am combining network and spider to produce NETSPID for a person that creates and analyzes networks. Finally the further historical development of triads in network science during the subsequent decades is out of the scope of this paper.

[17] During the Sunbelt XXXI, Annual Conference of the International Network for Social Network Analysis held at the Trades Winds Island Resorts, St. Pete Beach, Florida during February 8-13, 2011, I asked leading scholars such as Katherine Faust expert in triads, the main speaker of the event, Kathleen Carley and Elisha Peterson about the use of angle measurements and angle visualization and they expressed absence of such perspective in Network Science. In the particular case of Prof. Faust she shared that it was not a proper form of analysis within Graph Theory but subsequently accepted being open to receive this paper. Elisha Peterson specifically expressed he considered angle measurement was contrary to Graph Theory and that I should research more on topological studies not network graphs. Kathleen Carley demonstrated the most openness and interest in evaluating and considering angle

Nevertheless, when and odd number of components or nodes greater than one is involved in a conflict angles form generating a form of triangulation.

The justice system is rich in triad relations, conflict generally arises between two people and later a law enforcer intervenes, or the two actors have a relation that results in a conflict after a third actor enters into the relation, in judicial proceedings we regularly have the judge and the parties that will vary depending on the civil or criminal nature of the facts. If the matter is of civil nature the parties may be plaintiffs, defendant and judge or jury. In the criminal context we would have the prosecutor the defendant and the judge or jury. The procedure can get more complex if we have counterclaims, third party complaints, interpleader and class actions, but as will be developed in this paper, triads are the essential unit to evaluate more dynamical proceedings.

## II. MATHEMATICIAN LEONHAR EULER, VERTICES, EDGES, EVEN AND ODD NUMBERS AND THE SEED OF NETWORKS AS A SCIENCE

Leonard Euler (1707-1783) the most prolific mathematician in history,[18] in 1736 wrote a paper in which he solved a problem that originated in Konigsberg, eastern Prussia that gave life to graph theory and networks. The issue consisted of solving: "Can one walk across the seven bridges and never cross

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measurements in network analysis and visualization as all the other assistants to the conference. I also had the opportunity to sit down with Valdimir Batagelj and Anuska Ferligoj to discuss this approach and others related to balance or imbalance graphs based on the even or odd number characteristics of its vertices or edges, as well as using a more elementary unit to a block, the triangle. A block can be divided by two triangles and as such they could subdivide their blockmodeling methods and techniques with two triangles in every block to capture structurally and visually triads and all the theory that exist as well as the binary relation of even or odd numbers, by relating the triangles with odd and the block with the even structural nature of phenomena. Dr. Patrick Doreian expressed he had not considered this perspectives with blockmodeling and that he was willing to entertain it but it would have to be called something different. With regards to the approach on even and odd number to balanced or imbalanced graphs Arnout van de Rijt, Freeman Award Lecture speaker also enjoyed and was open to this new approach. Finally, Frank E. Tutzauer, Jamie F. Olson, Kathryn Coronges, Lukas Zenk, Schumpeter Tamada, Joe Parry, Julie M. Birkholz and Susan Lyons between others, all presented interest in the perspective. I am very thankful for their time to hear ideas of an outsider being them long time insiders in Network Science.

[18] E.T. BELL, MEN OF MATHEMATICS, THE LIVES AND ACHIEVEMENTS OF THE GREAT MATHEMATICIANS FROM ZENO TO POINCARE, (1937 reprint 1986)

the same one twice?" In solving this issue he simplified the problem by graphing the topography using vertices (points) for each lot of land and edges (lines) for each bridge. Once he graphed the topography he could visualize it and think how to solve it. Euler found a solution to this issue using the structural characteristics of even and odd numbers.[19]

Euler proved that there is no path crossing all seven bridges only once since nodes with an odd number of links must be either the starting or the ending point of the journey. A continuous path that goes through all bridges can have only one starting and one end point. Thus, such a path cannot exist on a graph that has more than two nodes with an odd number of links. This result depended on the structure of the situation, depicted ingeniously by the graph and the pure number theory using the structural nature of even and odd numbers, not another, more abstract, intuitive and multidimensional form of analysis.

In a recent paper I explored how we can decipher most types of phenomena through the contribution of Euler of "dividing and conquering" creating one-to-one relationships of actors and relations in social dynamics with numbers and then breaking them into two classes, even and odd.[20] An even number is a whole number that is twice another whole number, while an odd number is not divisible by two.[21] These are two characteristics of all natural numbers; they can be divided into the set of numbers that has no remainder after dividing by

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[19] Linked supra 9-13; see also for this graph: R. R. KADESCH, PROBLEM SOLVING ACROSS THE DISCIPLINES (Prentice Hall, 1997).

[20] Orlando I. Martinez-Garcia, *The Person in Law, the Number in Math: Improved Analysis of the Subject as Foundation for a Nouveau Regime*, American University Journal of Gender, Social Policy & the Law, Volume 18, (publication in process)

[21] THE PENGUIN DICTIONARY OF MATHEMATICS 151, 306 (David Nelson ed., 3d ed. (2003)

two, and the set of numbers that have a remainder of one after dividing by two. This method reduces an infinite collection of things to a finite collection.[22]

In addition, numbers are sets of things with order, structure, and quantifiable properties. Numbers' structure emanates from the operations of arithmetic that can be performed among and between them. In the quest for structures of phenomena, such as our previous Euler example of the Konigsberg Bridges, addition is an indispensable tool that gives us a unique integer for every pair of integers added together. We can classify this pair of integers as well as the unique product of their sums as even or odd. In similar fashion to the infinite set of integers, we can apply numbers to other problems in C2, law, economics and social dynamics, just like we have just described with even and odd numbers. The pattern that arises out of these arithmetical applications consists of the following three modalities: 1. the sum of any two even integers gives us an even integer. EVEN + EVEN = EVEN; 2. The sum of two odd integers gives us an even integer. ODD + ODD = EVEN; and 3. The sum of odd and even integers gives us odd integers. ODD + EVEN = ODD.

If we apply this scheme of unique products of addition of even and odd numbers to a person's behavior and social dynamics, we may predict the patterns of her effects. This method is remarkable, no matter how we label the two categories, be it with integers of even and odd numbers, or with colors such as black and white, or as here with types of social classes, the addition

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[22] "Mathematicians call this the group 'Zee-mod-two,' and denote it as Z/2. The letter Z denotes the integers; the 2 denotes the modulus-all numbers divisible by 2 are equivalent, and all integers having remainder of 1 after divisions by 2 are equivalent." JOSEPH MAZUR, EUCLID IN THE RAINFOREST, DISCOVERING UNIVERSAL TRUTH IN LOGIC AND MATH 105 (2005). In other words, this becomes a binary classification of a first group of equivalent integers under 0 and the second group of equivalent integers under 1.

would have the same results. This method shares the strength of binary analysis that so efficient has resulted in software programming. The only difference would be the names; hence, the implications of this methodology using numbers for persons would constitute a substantial leap in standard C2 reactions to crisis.

In order to apply this methodology to the issues regarding C2 dynamics, we should not commit the sin of oversimplifying. I will try to "keep things simple but not simpler,"[23] hence we would need to go deeper in identifying the even and odd structural characteristics and threshold levels of each type of actor or cluster. Once we make this codification, we can review the results by reiterating this application, in order to uncover structural characteristics, behavioral patterns and limitations just like Euler did with the Konigsberg Bridges and we hope to uncover through network science in C2 with the OM Model.[24]

## III. **Dyads and two body problems vs. triads and three body problem: methodological modern approaches to multiple actor dynamics in network science**

In math a triad forms a triangle and triangles are the elements of the simplest of the regular solids, the tetrahedron. The tetrahedron has four triangular faces, every face is an equilateral triangle, every vertex of a tetrahedron is incident to three faces and a three dimensional model of the tetrahedron is equivalent to a pyramid over a triangle base, not a square base like the real pyramids of

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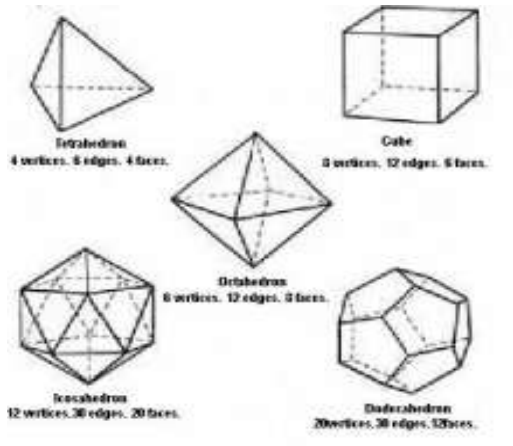
[23] Albert Einstein

[24] I have been tinkering with the nature of even and odd numbers and their correlation with symmetric and asymmetric, stable and unstable, static and dynamic phenomena. The present proposal may very well hold the key for the independent components of legal analysis as a new paradigm based on static, comparative static and dynamics as suggested by Peter Ziegler in *A General Theory of Law as a Paradigm for Legal Research*, 51 MOD. L. REV. 569, 589-592 (1988). I have personally approached the patterns from the perspective of Complexity theory, which is the underlying structure upon which other scholars have developed their theories. I have also developed a solution to the grand issue of when a small or large network is balanced or imbalanced based on this method. The product of such tinkering will be the subject of a separate paper.

Egypt and Mexico. The other four tridimensional solids known are the Cube, Octahedron, Dodecahedron and Icosahedrons, all regular polyhedral known as the Platonic or regular solids.[25]




Unlike planar figures, which could have infinitely many sides of equal length, the three-dimensional solids are limited to just these five.[26] However, two dimensional planar figures and these three dimensional solids share a common pattern that ends in the even number two. The common pattern was discovered by Leonhard Euler, who developed a proof by induction known as the *Euler characteristic* based on a planar two dimensional space, such as the Konigsberg Bridges graph previously related that supports the unique and

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[25] MORRIS KLINE, MATHEMATICS FOR THE NONMATHEMATICIAN 130 (1967). The *cube* has six faces, each face is a square and every vertex is incident with three faces; the *octahedron* has eight faces, each face is an equilateral triangle and every vertex is incident with four faces; the *dodecahedron* has twelve faces, each face is a pentagon and every vertex is incident with three faces; the *icosahedron* has twenty faces, each face is a triangle and every vertex is incident with five triangles.

[26] EDWARD B. BURGER & MICHAEL STARBIRD, THE JOY OF THINKING: THE BEAUTY AND POWER OF CLASSICAL MATHEMATICAL IDEAS, COURSE GUIDEBOOK, THE TEACHING COMPANY 43, 48 (2003). (A method of proof on only five regular solids involving the triangle as their building block is presented. First it is clarified that each face of a regular solid is a regular polygon and that every vertex has the same number of edges and faces incident to it and proceeds as follows: 1. Suppose we specify that each face of the solid we are going to construct must be a triangle. Could we construct a regular solid in which six equilateral triangles meet at each vertex?; 2. In the *tetrahedron*, three triangles meet at each vertex. In the *octahedron*, four triangles meet at a vertex, and in the *icosahedron*, five triangles meet at a vertex; 3. If we attempt to make a solid in which six equilateral triangles come together at a vertex, we see the impossibility, because those triangles will lie flat. Given that each angle in an equilateral triangle is 60 degrees, the total six triangles will be 360 degrees. A second proof with each face being a square is discussed but since a square can be subdivided into its simplest part, a triangle, I won't repeat it here)

exclusive group of the five Platonic solids based on triangles in our three dimensional perception of the World. The *Euler characteristic* consists of the following equation: V - E + F = 2. The way to put it into practice is as follows: First, draw a random shape with or without your eyes closed but without lifting the marker off the paper. Second, mark a dot at each point where the line drawn crosses itself and at the beginning and ending points. Third, count the dots (V for vertices) marked, subtract the edges (E for edges) between the dots, and add all the regions of the drawing (F for faces), the result will be two (2). When this two dimensional planar method is imported into three dimensional space represented by the five Platonic solids, the equation again coincides producing a two (2). The importance of this constant of two across random shape graphs in two dimensional planar graph and three dimensional solids cannot be emphasized enough.[27] Hence, we can expand on the force and structural patterns of triads as visualized through triangles in two dimensional planar graphs that directly depict structural common traits of three dimensional solids, of which the elementary structure is the tetrahedral which in itself has as elementary structure the triangle.[28]

In Network Science with triads we can begin to consider what is known as cliques and Bi-components. A clique is a maximal complete sub-network

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[27] Id. 48-49

[28] Id. 53-61. A single vertex (point or dot) is equivalent to zero-dimensions, an edge (line) with two vertices has one-dimension, a triangle with three vertices has two dimensions and a tetrahedron with four vertices has three dimensions. If dimensions are degrees of spatial freedom that might be infinite existing methodologies and our senses, with reasonable degree of certainty and experience perceive only three dimensions that can be corroborated with others directly or through scientific developments. The fourth dimension is beyond our daily experience and has predominantly been related to time and others to color but no consensus exist, these speculations are intuitive and uncertain. (See: PALLE YOURGRAU, A WORLD WITHOUT TIME, THE FORGOTTEN LEGACY OF GÖDEL AND EINSTEIN (2005)). I want to avoid intuitive conclusions that are unverifiable, unfalsafiable and based on analogies. Hence, my emphasis in focusing in DiaGrids, triads, triangles, tetrahedral and three dimensional space as simplest and most perceived scope of natural phenomena from which to build upon any analysis of any phenomena within the boundaries of such World. In specific, when the purpose of developing a method in this paper is to seek an interdisciplinary legitimate and reliable approach to identify and judge the actors responsible for the economic crisis. In this spirit we hope to surpass the pseudoscientific limitations that discretion in determinations of facts and hermeneutics in the meaning of the law lend themselves to, at the mercy of the leading actors of the system.

containing three vertices or more[29] while a Bi-component is a component of a minimum size of three that does not contain a cut-vertex.[30] It is also when a network has three vertices that relations become substantially more complex and dynamic as it has been illustrated in physics by the three-body problem.[31]

In order to determine if a network such as the solar system has a series that is balanced or that is imbalanced and hence its series diverges, the calculations would be infinite in large networks such as the solar system. The method used would be to complex, inefficient and hence "impossible." Similar to the issue of the Konigsberg Bridges that had entertained or wasted the time of many people trying to solve it, the three body problem was also openly brought up as an issue on the stability of the solar system in the following two mode way: Either the planets are to follow faithfully their orbits around the Sun forever, endlessly retracing the same path, in which case the Solar system is stable, or, because of the cumulative effects of gravitational perturbations, planetary orbits are destined to change radically in the distant future, completely altering the organization of the solar system, in which case the situation is unstable.[32] Does this read similar to the issue of balance or imbalance networks? Similar to social networks our Solar System is composed of actors and relations.

Our Solar System is technically composed of the Sun and all the objects that are attracted and held by the Sun's gravity. This includes the eight planets and

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[29] Id. 318 Pajek
[30] Id 318 Pajek
[31] TRINH XUAN THUAN, CHAOS AND HARMONY, PERSPECTIVES ON SCIENTIFIC REVOLUTIONS OF THE TWENTIETH CENTURY 73-75, 82, 347(2006)
[32] Id. 75, The exact phrasing of the issues was as follows:

> Given a system of arbitrarily many mass points that attract each according to Newton's law, under the assumption that no two points ever collide, try to find a representation of the coordinates of each point as a series in a variable that is some known function of time and for all of whose values the series converges uniformly.

many other celestial objects,[33] it seems we have a dyadic stable network composed of the Sun as a leading actor and all the other objects attracted to it. Besides the Sun the other existing actors compose a cluster of eight planets, an even number and as such using Euler's method of even and odd numbers we would have a balanced network composed of a dyad between the Sun in one part of the edge and the eight planets at the other end, cluster that is balanced. However, in order to make a more rigorous analysis regarding the balanced or imbalanced nature of our Solar System we would need to surpass the qualitative classifications of planets *vis* a *vis* other objects and take a more structural approach in which we count all these actors, planets, asteroids and other substantial objects, in order to count them and see if the end result is even or odd. If odd, the cluster is imbalanced and dynamical but not to a threshold level that makes them scale up to a leading actor, as the Sun or the whole cluster of planets and other objects. Hence, under this analysis our Solar system is balanced until a third leading actor intervenes into a triadic relation with the existing dyad. Thus, it is inherently important to explore, quantify and measure our Solar System in order to visualize our solar network and hence understand its stages and in what path is heading. This same approach is what is proposed for our social dynamics, the composition of our justice system and legal analysis.

Based on this method and for the sake of finding certain comfort, we would foresee that our Solar System has local-micro infinite imbalances within the Sun and throughout the cluster of planets and other objects, however in order for those imbalances to reach a threshold that amounts to a third leading actor with power enough to convert the existing dyad into a triadic relation is not

[33] Some of those other objects include asteroids such as: Ceres orbiting between Mars and Jupiter, Pluto that is no longer classified as a planet, Charon, 2003 UB 313 and others still under investigation within the Asteroid Belt and the Kuiper Belt. See http://www.universetoday.com/43827/our-solar-system/ (February 17, 2011)

common in the absence of any emergent actor or foreign visiting actor to our Solar System. In social-economic terms it would be analogous to the 20/80 rule or the Sun being a single body compared to the 20% elite and 80% of the social masses being the cluster of planets and other objects, the same social elite clusters throughout history have been pretty stable unless a third leading actor enters the scene to change the dyadic dynamics into triadic and then social dynamics become imbalance and more dynamical. Therefore, finding this commonality and this personal reflection as to how to solve the balanced-stable or imbalanced-unstable nature of our Solar System, how was the issue historically solved?

Subsequently, the issue posed of the three body problem was solved by Karl Friti of Sundman (1873-1949) in 1912 and was generalized to large networks of bodies by Quidong Wang.[34] Based on the previous experiences with triad, triangles and tetrahedral in three dimensional space that we best perceive based on our natural and biological sensory system, I find more reliable theories based on our three dimensions and less on theories based on metaphorical intuitive dimensions. The development of fractal theory supports that what we perceive in our three dimensional perception has bounded infinity, self replicating and scaling characteristics. Hence if what we perceive in three dimensions is self replicating to infinity we may just rely on that three dimensional perception that our experience throughout history has best perceived and used efficiently. Hence, that is a method that if it works in our three dimensional perceived environment it should also efficiently work in other infinite settings that are self-replicating in structure but at different scales

[34] Quidong Wang, *The Global Solution of the N-Body Problem*, Celestial mechanics and Dynamical Astronomy, Vol. 50, No. 1, p. 73-88 (1991) See also: http://en.wikipedia.org/wiki/Henri_Poincar%C3%A9#The_three-body_problem (February 19, 2011)

despite our biological and technological limitations to perceive them as we perceive three dimensional phenomena.

With this background we now evaluate the existing network science techniques used to study triads as originally developed by George Simmel in his three classical functions of third actors as: Mediation, *Tertius gaudens* and Despotism.[35] These three functions were later expanded by others[36] to cover other roles such as: Brokerage, Coordinator, Gatekeeper, Itinerant broker, Liaison and Representative. Nevertheless, these qualitative classifications share the same flaw confronted by mathematicians with the three body problem, they rely on classical analysis fit for dyads analogous to the two body problem under Newton's approach, which as previously described was inappropriate for the more dynamical three body problem equivalent to triad analysis for which a new approach is needed. The main flaw is that already explained of assuming fixed causal relations, that if B is a friend of C and A is a friend of B then A is a friend of C. This transitivity law can break down as soon as A meets C. For example, assume A and B are good friend and C is a beautiful girl that B likes. As soon as A meets C, if C shows any inclination or signals any attraction to A or at least B regards or believes some inclination or signal has existed, his friendship with A may break down. B might get jealous of A or simply feel threatened by A and might cut off the friendship immediately if B has substantial interest in C. Thus, the relation between A and B will not be

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[35] *Mediation* is the third element that mediates conflicts and facilitates negotiations. This assumes both members perceive him as sufficiently impartial. *Tertius gaudens* is the third element that exploits the conflictual relation between two comparable power bases. If each base attempts to enlist her into alliance against the other, she can play both sides, raise the stakes and win out against both when they run out of ammunition. Finally, *Despotism* is the function of 'divide and conquer'. The third element instigates conflict between the other two so as to preclude any alliance against herself. The resulting split underpins her power base, but precariously so, because it vanishes as soon as the split heals. Id. 120-121.

[36] Gould, R.V. and Fernandez, R.M., *Structure of mediation: a formal approach to brokerage in transactions networks*, Sociological Methodology, 89-126(1989)

stable and fix, it can vary. It is this type of flaw that tends to push our analysis of balanced or imbalanced networks to a more structural approach based on the number of actors or relations, if the numbers of actors are three, we have an odd number and hence somebody is left out, either A or B get C at certain time and space and hence this network will always be imbalanced structurally. Triad and Coalition studies developed by George Simmel, Theodore Caplow, W.A. Gamson and R. Burt hint that when a triad exist there is an imbalance when they support that "the mere presence of a third element, however passive, tends to modify the original dyadic relation. This is why a triad can never be reduced to the sum of three dyads."[37] However, these authors fail to grasp fully the imbalance and do not provide a method to tackle this issue as I do with the use of even and odd nature of actors in the network.

Finally, I would add that the holy grail of analysis of networks is not triads, but rather the structural characteristic of a network being of an even or odd nature. Hence in such analysis dyads as an even structure and triads as an odd structure are merely different stages in time and space as networks evolve being both holy grails of network science. Network Science has a need to focus research in understanding, measuring and visualizing how even structures evolve into odd structures and vice versa to infinity as fractal theory teaches us. In that framework we should study Random, Small World, Scale free and others yet to be explored types of networks.[38] In the study of the various

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[37] ALAIN DEGENNE AND MICHEL FORSE, INTRODUCING SOCIAL NETWORKS 122, 119-130 (1999 reprinted 2004).(Explaining in a concise and simple manner the how triads are the holy grail of network analysis as first identified by George Simmel with his writings on triads and coalitions and how others developed it further, such as Theodore Caplow, W.A. Gamson and R. Burt.) See also *Degnenne* and *Forse* citing T. CAPLOW, TWO AGAINST ONE: COALITIONS IN TRIADS, (1968), R. BURT TOWARD A STRUCTURAL THEORY OF ACTION (1982) and STRUCTURAL HOLES: THE SOCIAL STRUCTURE OF COMPETITION (1992a); W.A. Gamson, *A theory of coalition formation*, American Sociological Review, 26(3); 373:83(1961), id.

[38] See: ALBERT-LASZLO BARABASI, LINKED, HOW EVERYTHING IS CONNECTED TO EVERYTHING ELSE AND WHAT IT MEANS FOR BUSINESS, SCIENCE, AND EVERYDAY LIFE (2003), SIMON REAY ATKINS AND JAMES MOFFAT, THE AGILE

transitions from even to odd networks existing measurement methods known as degree distribution, clustering coefficient, average path length and types of preferential attachments are essential in understanding how networks grow and evolve over time.[39]

## IV. Classical degree measurement and network science, a new perspective[40]

The dynamical progressive path of a natural phenomena as depicted in the Period-Doublings and Chaos diagrams implies discrete bits of progress and stages in sequence that can be depicted with the nature of even and odd numbers if applied to dynamical network analysis. I propose the incorporation into network science of classic mathematical analysis of one vertex (ego), two vertices (dyad) and three vertices (triad) dynamics. The proposed amendment is simple but revolutionary. After these three levels of analysis are established a foundation for complete large macro level network analysis becomes available.

The second contribution would be the revival and implementation of using even and odd number analysis as networks develop from single vertex to triads and get larger. As previously illustrated even and odd number theory help identify when a graph or system is balanced or imbalanced. When we don't have a vertex, equivalent to a zero, the nature of the system is neutral, when we have

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ORGANIZATION, FROM INFORMAL NETWORKS TO COMPLEX EFFECTS AND AGILITY 97-102 (2005 reprinted 2007)(Explaining differences and characteristics between Random, Scale Free and Small World networks)

[39] SIMON REAY ATKINSON & JAMES MOFFAT, THE AGILE ORGANIZATION, FROM INFORMAL NETWORKS TO COMPLEX EFFECTS AND AGILITY 42-48,(2005, 2nd Printing September 2007)

[40]As part of the paper titled *Strategic Network Perspectives for Unstable Regions Through the Justice System*, presented at the 5th Annual Network Science Workshop organized by the Network Science Center at West Point Military Academy, New York, during the 26, 27 and 28 of October, 2010, I consulted with Dr. Vladimir Batagelj co-author of EXPLORATORY SOCIAL NETWORK ANALYSIS WITH PAJEK, (2005) about the use of angle measurement and he confirmed that he had not seen network visualizations based on measurements of angles even though he has implemented in Pajek software the identification of the closest angle of a whole graph and triangles for short cycle connectivity. See. V. Batagelj, M. Zaversnik, *Short cycle connectivity*, Discrete Mathematics 307 (2007) 310-318. During the workshop I asked participants, such as: John M. Graham, Chris Arney, Zhen Wen, Dashun Wang, John James, Henry Hexmoor, Paul Stanton, Lynndee Kemmet, Mark Thomas, Guenther Kress, Josh Lospinoso and others present about this approach in search of feedback and all confirmed the novelty of it as proposed within "network science".

one vertex, we have an odd number and the system is imbalance towards one vertex. If we have two vertices, a dyad forms and the relation is balanced in terms of one or the other, we can easily predict in which vertex the resources or power is resting. The power or resource relation complicates with three vertices, and odd number that is dynamical due to the structural imbalance odd numbers depict. Ironically odd structures are very stiff and strong as the triangles in the DiaGrid system prove but this strength is the product of stress and energy generated by the triad odd imbalanced dynamic, which achieves through competition and uncertainty maximal exploitation or performance of the actors.

This type of balance and imbalance of relations based on even or odd number of actors and relations has not been exploited in network science nor the law or the justice system which has focused on positive and negative relations, vertices and edges.[41] The use of even and odd analysis solves the existing issue regarding the proper efficacy of analysis based on balance and imbalance relations made in the previously discussed three levels of analysis (ego, dyad and triad) and once networks evolve and become complex.[42] The extension of the even and odd number of actors applies to large networks because the essence and characteristics that make numbers even or odd extend to infinity. Furthermore, this method helps identify the network's dynamical or stable structure based on pure structure and not subjective demarcations of positive or

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[41] In the law I have proposed the use of even and odd numbers and I have found that such proposal is implicitly supported by RICHARD A. POSNER, ECONOMIC ANALYSIS OF LAW 20-23, Sixth Edition (2003). (Illustrating samples of game theory applied to economic analysis. The illustration relies on presenting scenarios of a monopolist setting prices if ten prospective entrants to a market exist, a person with six shots and ten assailant, twenty five lions and one lamb on an island. Under all three scenarios Posner suggest that the number of actors involved would determine much of the strategy to follow and/or outcome. Posner specifically expresses in the problem of the lions and the lamb if the answer would be different if there were 24 rather than 25 lions? Here Posner is basically playing with the method of even and odd numbers without ever developing its potential as in done in this and another paper I previously published, supra footnote 20)

[42] See JOHN SCOTT, SOCIAL NETWORK ANALYSIS, A HANDBOOK 7-16 Second Edition (1991 republication of 2007)

negative relations. Finally, these novel contributions epitomized in the enhanced network science method herein identified as the Orlando-Manatí Model (OM Model) provides an efficient alternative bottom up approach to C2.

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