Predicting Placement into MA101
Which assessments should we use to place students into the developmental math class at the Prep School?

Greta Holtackers

This paper was completed and submitted in partial fulfillment of the Master Teacher Program, a two-year faculty professional development program conducted by the Center for Faculty Excellence, United States Military Academy, West Point, NY 2013.

ABSTRACT

The purpose of this study was to establish the best measurement(s) for determining student placement into MA101 Algebra/Trigonometry, the lowest level mathematics track at the Prep School. I investigated four measurements: math SAT scores, math placement exam scores, high school experience rating, and assessment averages from the first half of the first quarter of Pre-Calculus (considered the standard track). My sample data consisted of student scores from this academic year (AY12-13). To analyze the data, I looked at the relationship of each measurement with actual student placement in each track (Algebra/Trig, Pre-Calculus, and AP Calculus). I also calculated the sample mean and standard deviation for each measurement for the class as a whole, as well as for each track. I then examined the Pearson correlation coefficient for each measurement and how many students would have been placed in the incorrect track if we had only used that measurement as a basis for placement. The study revealed that the first four-week assessment averages were the best predictor for placement into the Algebra/Trig track.

Last week a student asked to speak to me privately after class. He was worried about his SAT scores and their implication regarding his acceptance to the academy. He had taken the SAT several times in high school and had scored in the 400’s in math each time. Because he was determined to go to West Point, his parents enrolled him in an after-school tutoring program designed to increase his scores. His SAT scores in English and on the writing sample were very good, and since West Point only records the highest score from each section, the tutoring agency advised my student to skip everything except for the math portion the next time he took the SAT. Their reasoning was that he could score low on the other sections; therefore they advised him to concentrate only on doing well on the math portion of the exam. For months he prepped for the math section of the SAT exam, and on his next try he scored 100 points higher in math (he scored the minimum score for every other section). Obviously, since he was in my class, this student was not a direct admit to West Point. His SAT scores met the standard for admission to West Point, however, because he was a prior service soldier he was sent to the Prep School. This student had come to talk to me because he was nervous about having to take the SAT again, a requirement for every student at the Prep School. My reason for including this story as part of this paper is that this student was not placed into Algebra/Trig until the second quarter (placement normally occurs at the midterm of first
quarter). His math instructor had chosen to keep him in Pre-Calculus despite his grade for the first four weeks of the first quarter because his math SAT score was above average (for the Prep School). Not until it was obvious that he didn’t have the fundamental skills to be successful in Pre-Calculus was he placed into the lower track. This student is a prime example of how misleading SAT scores can be when deciding student placement in math classes at the Prep School.

The Prep School is a one-year program designed to prepare candidates selected by the United States Military Academy Admissions office for the academic, physical, and military rigors of West Point (http://www.usma.edu/USMAPS/SitePages/Home.aspx). The math department at the Prep School is comprised of three tracks: AP Calculus, considered the highest-level track, Pre-Calculus, considered the standard track, and Algebra/Trig, considered the developmental track. Since this paper is focused on the placement of students from the Pre-Calculus track into Algebra/Trig, I will only talk about the primary goals of these two courses. Pre-Calculus is designed to prepare students for the standard freshman level math courses at West Point while also challenging students to develop skills past the basic requirements for admission into West Point. The Algebra/Trig course is a rigorous course designed to help students remediate missing fundamental skills, as well as prepare them for the standard freshman level math courses at West Point.

Five years ago the math classes at the Prep School consisted of AP Calculus, Standard Math and Repeat Math. We used SAT math scores, the placement exam scores, and the survey results only for placement into the AP Calculus track at the beginning of the year. Every other student who wasn’t placed in AP Calculus was placed in Pre-Calculus. A student was only dropped down to Repeat Math at the end of a quarter if he/she finished that quarter with a grade below 70%. Repeat Math consisted of exactly the same syllabus and assignments as the previous quarter in Pre-Calculus. Essentially, the students simply repeated the quarter that they failed. There were significant problems with this model. Specifically, if a student was obviously struggling in Pre-Calculus he/she had to wait until the end of the quarter to be moved into the appropriate course. And yet, that “lower” course didn’t address the needs of those underprepared students since it simply repeated the previous quarter. At the same time, because weekend privileges were linked to grades, we would have students who were capable of being successful in Pre-Calculus that would purposely fail a quarter so that the next repeated quarter was very easy for them resulting in a higher grade with less effort and more weekend privileges. I taught Repeat Math five years ago, and it was a challenge. The students that needed extra help were frustrated, and the students who were sandbagging were usually discipline problems (most likely due to boredom).

In response to the obvious issues with Repeat Math, the Prep School created the Algebra/Trig course involving establishing new goals and objectives, as well as creating a more focused curriculum designed at addressing the unique needs of under-prepared students. With this new course came the challenge of deciding which students should be placed in Algebra/Trig and which students should remain in Pre-Calculus. Because of our experiences in Repeat Math, we did not want to place students in Algebra/Trig who were simply looking for an easier class that would give them a higher grade point average and more weekend privileges. We also didn’t want to leave struggling students in Pre-Calculus for an entire quarter resulting in failing grades and demoralization. At the same time, we were well aware, from both experience and research findings, that SAT scores were not good indicators for who would need to be placed into the developmental track. Consequently, we decided to have Algebra/Trig start at the midterm of the first quarter and use instructor recommendations as a means for placing students into Algebra/Trig.
METHODOLOGY

The math department at the Prep School collects three scores from students prior to the start of the academic year; their highest score on the math portion of the SAT, their score as a percent on the department’s placement exam, and a high school experience rating of 2-6 based on the math courses that each student took prior to attending the Prep School. The placement exam is a no-technology, multiple-choice test consisting of 50 questions. The students are given an 80-minute time limit and are told not to randomly guess (as guessing could lead to a false representation of their fundamental skills). To determine a rough measurement for experience in mathematics we give every student a survey after they are finished taking the placement exam. The survey asks them to indicate which math classes they have taken, in what year of high school they were enrolled in that class, and what grade they remember earning in each class. The senior instructors analyze the responses to the surveys and give each student a rating between 2 and 6. A student is given a 2 if he/she has had only one algebra course (this is rare), a 3 if he/she has had two courses in algebra, a 4 if he/she has had two courses in algebra and some trigonometry, a 5 if he/she took Pre-Calculus, and a 6 if he/she had taken calculus before enrolling at the Prep School. These scores are primarily used to determine which students are qualified to be in the AP Calculus course. However, these scores are also added to a spreadsheet designed to assist Pre-Calculus instructors in determining which students should be placed in Algebra/Trig.

At the beginning of the first quarter, any student who was not placed in AP Calculus is put into a Pre-Calculus class. For the first four weeks the Pre-Calculus course is standardized so that each instructor gives the same quizzes and exams, and we record these scores on the same spreadsheet that holds the other three measurements. Our aim is to identify those students who truly lack fundamental math skills and place them in Algebra/Trig, while also leaving those students who are viewed as having the ability to perform well but are not because of a lack of effort in the Pre-Calculus track. Because the Prep School has the fundamental goal of helping students build a strong academic foundation, each math class meets for two periods every day and each instructor offers an optional additional instruction period four days a week. Consequently, instructors have a significant amount of contact time in which to get to know their students. i.e. they can observe students performing problems, they can gauge homework completion, and they can track attendance for the additional instruction periods. After four weeks of instruction, instructors make a recommendation for who, from their sections, should be placed in Algebra/Trig. Then, at the midterm of the first quarter (after those first four weeks), we determine the number of classes of Algebra/Trig that are needed based on instructor recommendations, and we give the students a new schedule. This method of placing students is somewhat subjective, and while many students are fairly easy to judge, there are always 10 to 20 students that we are unsure of. Once students are placed in the Algebra/Trig track, we don’t move them back up to Pre-Calculus if we feel they have been mistakenly placed in the lower track. However, after the first quarter and during the second and third quarters, if a Pre-Calculus instructor feels that a student should have been placed in Algebra/Trig, we move the student down to the lower track. Moving students into Algebra/Trig throughout the year has lead to issues such as over-sized classes and a lack of opportunity for those students to remediate under-developed skills. In an effort to create better guidance for the Pre-Calculus instructors, I analyzed the scores from this year’s class on each of the three measurements
taken at the beginning of the year, as well as each student’s average on the quizzes and exams given in the first four weeks of the first quarter.

After examining numbers of students within Algebra/Trig and Pre-Calculus with scores that, in the past, have been deemed as putting them at risk for being placed in the developmental track, I calculated several statistics in an effort to find the measurements that are the best indicators for placement into Algebra/Trig. I calculated:

- The percent of students with SAT math scores in a 20-point range that had been placed in each course.
- The percent of students with placement exam scores in a 10 percent range that had been placed in each course.
- The percent of students that were assigned a rating, 3 through 6 (there were no students with a rating of 2 this year) that had been placed in each course.
- The percent of students with first-half quarter 1 assessment averages in a given range that had been placed in Algebra/Trig and Pre-Calculus (AP Calculus students were not part of this data).
- Correlations between each measurement and placement into Algebra/Trig or Pre-Calculus.
- The number of students that would have been placed incorrectly if we had used only one measurement as a placement guide.

RESULTS

The class of 2013 consisted of 228 students. Forty-three students were placed in Algebra/Trig, 132 were placed in Pre-Calculus, and 53 were placed in AP Calculus. The SAT math scores of the entire student body ranged from 410 to 770. In the math department at the Prep School, instructors consider students with a math SAT score at or below 500 to be deep risk. There were 36 students that came to the Prep School this year with a math SAT score at or below 500. Of those 36 students, 19 were placed in Algebra/Trig and 17 were placed in Pre-Calculus.

The placement exam scores for this year’s class ranged from 8% to 90%. We consider students with a score below 45% to be at-risk. Of the 74 students this year that scores below 45% on the entrance exam, 35 were placed in Algebra/Trig and 39 were placed in Pre-Calculus.

There were 28 students this year that were given a high school rating of 3, meaning that while they had two courses in algebra, they had never learned trigonometry. Of those 28 students, 13 were placed in Algebra/Trig and 15 were placed in Pre-Calculus.

There were 38 students that earned a quiz/exam average below 75% in the first-half of the first quarter. Of those 38 students, 35 were placed in Algebra/Trig and 3 were placed in Pre-Calculus.

A more detailed breakdown of the percent of students that fell in a range in each measurement and their respective placement is displayed in graphs below.
RELATIONSHIP OF SAT SCORES WITH PLACEMENT

RELATIONSHIP OF PLACEMENT EXAM SCORES WITH PLACEMENT

RELATIONSHIP OF HIGH SCHOOL SURVEY SCORES WITH PLACEMENT
In an effort to compare the relationships demonstrated above, I calculated Pearson's correlation coefficient for each measurement. I assigned Algebra/Trig a value of zero and Pre-Calculus a value of 1.

**Correlation Between Measurement and Placement**

<table>
<thead>
<tr>
<th>MEASUREMENT</th>
<th>CORRELATION COEFFICIENT</th>
<th>TYPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>SAT</td>
<td>0.330446084</td>
<td>small</td>
</tr>
<tr>
<td>Placement Exam</td>
<td>0.560647180</td>
<td>medium</td>
</tr>
<tr>
<td>HS Survey Score</td>
<td>0.238588682</td>
<td>small</td>
</tr>
<tr>
<td>Q1 Assessment Average</td>
<td>0.779581482</td>
<td>strong</td>
</tr>
</tbody>
</table>

After looking at the correlations between each measurement and placement, I decided to ask the question of “what if?” In an effort to re-examine what we might consider to be scores that would put a student at risk for being placed in Algebra/Trig, I calculated the mean SAT score and placement exam score of the students in Algebra/Trig this year. The mean SAT score was 514 and the mean placement exam score was 35.6%. I kept the risk level of the high school survey rating at a 3, and the risk level of the first quarter quiz/exam average at below 75%. I then calculated the number of students that would have been placed in the incorrect course (i.e. students who belonged in Algebra/Trig that would have been placed in Pre-Calculus, and students who would have been placed in Algebra/Trig that could have been successful in Pre-Calculus) had we only used that measurement as a deciding factor.
Number of Students Who Would Have Been Placed Incorrectly Had We Used Each Measurement as the Single Deciding Factor for Placement

<table>
<thead>
<tr>
<th>MEASUREMENT</th>
<th>NUMBER OF INCORRECTLY PLACED STUDENTS</th>
<th>PERCENT OF INCORRECTLY PLACED STUDENTS IN ALG/TRIG AND PRE-CALCULUS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math SAT Score &lt;=520</td>
<td>55</td>
<td>31.3%</td>
</tr>
<tr>
<td>Placement Exam Score &lt;=36</td>
<td>31</td>
<td>17.6%</td>
</tr>
<tr>
<td>HS Survey Score = 3</td>
<td>44</td>
<td>25%</td>
</tr>
<tr>
<td>Q1 Assessment Ave &lt;= 75%</td>
<td>11</td>
<td>6.3%</td>
</tr>
</tbody>
</table>

**CONCLUSION**

The data indicates that math SAT scores and high school survey ratings are the poorest indicators for determining which students should be placed in Algebra/Trig. The placement exam has a medium level of correlation, but the strongest indicator for placement into Algebra/Trig is a student’s quiz and exam average from the first four weeks of the first quarter. In fact, there is no measure that could allow us to place students in the beginning of the year as accurately as waiting four weeks and using performance data to place students. In other words, the best indicator for future performance is current performance.

**RECOMMENDATIONS**

If we only use the average of the assessment scores from the first half of the first quarter, we still would have incorrectly placed 11 students. While not as many students would have been incorrectly placed as if we had used any of the other measurements, 11 students still constitutes an entire Algebra/Trig section and we would continue to have oversized Algebra/Trig classes and scheduling problems. Can we make the first quarter assessments an even better indicator? Are there other measurements that we could calculate in the first four weeks that would allow us to make more accurate placements? Could we improve the placement exam and use it as an additional measurement for placement? Should extreme scores on the other measurements, i.e. very low math SAT scores or a high school rating of 2, continue to be considered when making a recommendation for placement?

The challenge in the first four weeks of the academic year is differentiating between three types of students. The first type of student is the one who lacks fundamental skills in math but has such a strong desire to perform well that he/she will spend an inordinate amount of time studying, and will consequently perform pretty well in the first four weeks of the first quarter. However, we know from experience that as the material in Pre-Calculus becomes more involved, the grades of these students will most likely fall. We also find that while these students may perform well on the assessments in the first quarter, when they are given follow up assessments later in the year that measure fundamental skills, they do not earn a score that would support a claim that they have remediated their lacking skills. The second type of student is one that has the fundamental math
skills required to perform well in Pre-Calculus, but he/she lacks the work ethic and discipline to earn an A or B. From experience we know that if these students are placed in Algebra/Trig they will most likely not improve their study skills and continue to display a lack of effort. Thus, Algebra/Trig is not an avenue that allows these students to remediate the skills they need in order to be successful in math in the future. The third type of student in one who lacks fundamental skills, and is also lacking in his/her effort level, whether because he/she is struggling in class so much that he/she cannot do their homework, or because he/she lacks fundamental study skills and discipline.

**Recommendation 1)** The Prep School math department should continue to start Algebra/Trig at the midterm of the first quarter. There is no measurement that would allow us to place students in the beginning of the year as accurately as waiting four weeks and using performance data to place students. Since we have the flexibility in our schedule to be able to allow for this data to be collected, and since there are no clear ramifications for waiting four weeks to place students into Algebra/Trig, we should focus our efforts on improving the data we can collect for each student during the first half of the first quarter.

**Recommendation 2)** By making the quizzes and exams more difficult, i.e. more representative of the level of difficulty of Pre-Calculus throughout the year, we may be able to more easily identify the type 1 student described above.

**Recommendation 3)** Is there a way that we could measure the amount of time and effort that a student is putting into the Pre-Calculus course in the beginning of the year? We could consider daily or weekly time surveys. For example, each day we could have students record the amount of time that they spent studying math the night before. Or, we could ask each student to give an average amount of nightly study time at the end of each week. We could also ask for feedback from our students regarding their comfort level with the material. Each week we could give them a survey asking them to rate how well they feel they understand the concepts and skills that were taught throughout that week. Of course, along with taking these measurements, we will also have to determine how we should apply them and whether or not they will actually achieve the goal of giving us a picture of how much effort a student is putting forth towards understanding the material. If we can develop a measurement for effort and comfort level we may be able to better differentiate between the three types of student described above.

**Recommendation 4)** The placement exam was originally designed as an aid in identifying students who should be placed in AP Calculus. We could re-examine the design of the placement exam and use it as an additional measurement when identifying students who should be placed into Algebra/Trig. More specifically, we could break the placement exam into sections measuring different types of math skills such as basic algebra, geometry, trigonometry, advanced algebra and calculus. We could then not only use a total score as a measurement, but we could see how well students performed on each section and include information on the types of problems that a student skipped. By improving this measurement we may be able to better identify those students who may struggle more in Pre-Calculus as the year progresses.
**Recommendation 5)** Scores on the math portion of the SAT, and the high school survey ratings should continue to be taken into consideration, especially if those scores are significantly below or above average. These scores may allow us to better identify the three types of students described above. More specifically, if a student has an assessment average below 75%, but they are well above average in the other three assessments, we are most likely talking about a type 2 student. If a student has an assessment average in the B range, but his/her other measurements are significantly below average, we are most likely talking about a type 1 student.

**Recommendation 6)** More research should be done in measuring the accuracy of our placement decisions. We should survey the Algebra/Trig and Pre-Calculus instructors to see if they feel there are any students that should have been placed in the other course.