EDITOR'S NOTES

This issue is dedicated to Joseph Arkin: Founding Editor of *Mathematica Militaris* and an American Ramanujan . That Mr. Arkin is compared to the well-known, self-educated mathematician, Ramanujan, will beocme clear when you read our lead article "Joe Arkin: A Biography" by COL Arney:

> "Joe Arkin learned his high school mathematics at Brooklyn Boys High. He understood algebra and geometry, but had not studied trigonometry, calculus, linear algebra or analysis…he had taught himself methods of inquiry…"

Joe Arkin had a "great intuition" and a "tremendous feel" for number theory and eventually ent on to work with famous number theorists Paul Erdos and Ernst Straus. He should be an inspiration not only to the faculty at the service academies but also to our students. Mr. Arkin's accomplishments are a testimony to what determination and had work can do.

Several articles are dedicated to the study of computational fluid flow problems. The systems of interest to the military are geometrically complicated and the flow phenomena produced is complex. David Haroldsen provides an interesting overview of the recent strides made in the generation of the grids required for complex structures, in the accuracy and stability of numerical flow solvers and in the effective post-processing of data. Paul Weinacht and this editor along with USMA Cadet John Brengle present results obtained from a numerical investigation of lateral control jets for projectiles in supersonic flight.

Mathematical research can be varied and diverse. I call your attention to the Beaver, Matty, Phillips article for a flavor of the research opportunities at the Army Research Laboratory, at the Operations Research Center and others. LT Mielstrup provides an interesting article regarding the U.S. Coast Guard Academy's Operations Analysis Course.

One of our main goals is to inspire our students. Many of them do not realize the doors that mathematics opens, not only doors of opportunity for successful careers but a door that opens the mind to different ways of thinking and to gain a better understanding of how things work. Let's hope that we are doing our best to help inspire the future mathematical researchers for the military.

Best wishes from West Point,
Mary Jane Graham

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Department of Mathematical Sciences
United States Military Academy
ATTN: MADN-MATH

West Point, New York 10996

JOSEPH ARKIN: A BIOGRAPHY

COL. David C. Arney, USMA

This issue is dedicated to Joseph Arkin: Founding Editor of *Mathematica Militaris* and an American Ramanujan

Biography by David C. Arney (USMA): Joseph Arkin joined the Mathematics Department at West Point in 1986 "to do math." Joe arrived at the Academy with his wife Judy. Together they had raised a family of four girls, Helen, Jessica, Aviva, and Sara. There was no doubt that mathematics was and always will be his true passion. He was deeply in love with the subject. Yet, Joe wasn't a typical mathematician. He was a self-educated, amateur mathematician, completely dedicated to "doing" mathematics.

Joe Arkin learned his high-school mathematics at Brooklyn's Boys High. He understood algebra and geometry, but had not studied trigonometry, calculus, linear algebra, or analysis, although he definitely knew methods of proof and logic. On the other hand, he had taught himself methods of inquiry very well. There was absolutely no mathematics problem that he wasn't deeply interested in. He had great intuition and tremendous feel for his forte, number theory. He had a new idea everyday, a new problem to investigate every week, a new direction to pursue every month. By 1986, Joe had already published 25-30 articles in prestigious journals and had worked with several prominent mathematicians.

Joe had a driving desire to select West Point for his mathematical home. For several years, he had been a mathematical independent, with an affiliation with the New York Academy of Sciences, and partnerships with several mathematicians. His desire to work at the Military Academy came from his experience as a soldier in the Army during World War II. To Joe, doing mathematics at West Point was like serving his country as he had done 40 years before. Joe and Judy made the 25-mile, 35-minute car trip from Spring Valley to West Point, hundreds of times over the next 10-12 years.

His first mathematical inquiry at West Point was tiling the plane using rectangles of sides with lengths of Fibonacci numbers. His work on that problem eventually led to several papers and presentations. Joe didn't know all about higher mathematics, but he sure knew numbers. To Joe, every number was special, every pair of numbers shared some properties, and every set of numbers were connected, somehow, someway. Tiling using Fibonacci numbers was a nice geometrical use of a sequence of numbers with special properties. After that project, Joe and his West Point colleagues took on the challenge of arranging numbers in 2- and 3-dimensional arrays, squares and cubes, with special (called magical) properties. Joe Arkin was a master at this kind of number theory. At first, Joe was not a fan of computers. In his mind, they were hindrances to "real" mathematical thinking. Computers couldn't do Joe's mathematics of conjecturing, analyzing, and proving. Joe had a gift for finding patterns. In Joe's mind, numbers came alive, and he visualized the patterns and properties that were hidden to others. Eventually, Joe began to use computers to produce numbers that came from exhaustive iteration or complicated formulas. Joe was always in need of large prime numbers, and the computer was able to produce what he wanted very quickly. Eventually, theoretical mathematician Joe Arkin became a serious computer user. Joe's West Point number theory team constructed all sorts of magic squares and cubes with special properties. One special cube was called the Cameron Cube, named for the department head at West Point. Another one was named the Supercube.

Joe knew everyone in the number theory community. He never forgot the people that he met. He could tell stories about people that he had met only once at a conference 20 years before. He remembered the significant parts of their mathematics. He just enjoyed meeting and talking with and about mathematicians. He was kind, generous, and concerned about other people. He loved people and, in return, everyone loved Joe.

Joe's mathematical career started in the early 1960s. Joe was doing mathematics on his own and wanted to meet other mathematicians to share ideas. He had written a 30-page paper on a topic related to power series and their properties. After some connections were made, Joe visited F. A. Ficken at New York University. Ficken gave Joe's paper to Richard Pollack, who suggested that Joe rewrite the paper, making it shorter and more concise. Pollack suggested sending the paper to Leonard Carlitz, a number theorist at Duke. Under Carlitz's guidance, the paper was reworked, reduced to 2 pages, and published in the *American Mathematical Monthly*.

The Arkin family began a tradition of driving to the summer meetings of the American Mathematics

Society, no matter where they were in America -- San Jose, Vancover, Columbus, Laramie, Missoula, Phoenix. A couple trips were taken across the entire country in the late 1960s and early 1970s with a 17-foot trailer. The Arkins were sightseers, who enjoyed their family vacation; and travelers, trying to arrive at a professional mathematics meeting to present state-of-the art research. Joe and Judy have many wonderful memories of these cross-country excursions.

Joe's first encounter with his close friend Paul Erdos came on a plane going to a number theory conference at Washington State University in 1971. This was Joe's first conference. Erdos was giving one of the principal addresses. Joe was one of 18 others, including his future friend and mentor Ernst Straus, invited to talk on their work. Paul Erdos and Joe were seated nearby on the plane, and Joe asked Erdos to read his paper entitled "Researches on Some Classical Problems." It was the start of their life-long friendship. Joe also had a productive and rewarding relationship with Ernst Straus. Joe met Straus through Paul Erdos while attending that conference. An accomplished mathematician, Straus, had worked with Einstein and was a number theorist. Joe Arkin and Straus became productive partners. While they are listed as co-authors on only two papers and Joe extended some of his work with Straus in a third paper, they were close collaborators and friends for a number of years.

Arkin, Straus, and Erdos were a strong trio of number theorists. Straus was Einstein's assistant at the Institute for Advanced Study in Princeton. Erdos was simply the greatest mathematician of the 20th century and the most prolific author of all time. The three men were colleagues when it came to thinking about number theory and doing math. Each gaining ideas and inspiration from the others. Mrs. Louise Straus, Ernst's wife, recalled this trio's relationship much later when she wrote to Arkin: "Both Erdos and Ernst were very supportive of your work, very encouraged and interested in your ideas. I know this for a fact because I often heard them talking about you and your work. If you were to mention the encouragement you received from both of them, you would be doing them a true honor." Joe's special relationship with his colleagues was recounted in a eulogy for Straus given by UCLA mathematician Albert Whiteman: "Ernst Straus was an avid collaborator. He wrote numerous joint papers and I now wish to speak very briefly about his work with two of his principal collaborators -- Paul Erdos and Joseph Arkin. Professor Paul Erdos is one of the world's most famous mathematicians, a grand master of mathematics. Erdos' visits to UCLA were marked by intense discussions between Ernst and Paul. The resulting interplay of ideas was both exhausting and highly productive. A joint paper was usually the outcome. It was my privilege to listen in on some of the discussions between these two mathematicians. It was awe-inspiring to hear them develop intricate arguments without putting pencil to paper or chalk to blackboard. Joseph Arkin, a disabled World War II veteran, was an amateur mathematician. Unlike Erdos, he seldom traveled. His joint work with Straus was carried out mostly by correspondence and long distance telephone conversations. Arkin is a highly original and imaginative mathematician, but he has great difficulty in expressing his ideas clearly and precisely. Ernst Straus gave him friendship, guidance and encouragement. Their collaborative efforts produced several beautiful papers on Latin Systems and Diophantine equations."

In the late 1960s, Joe Arkin was given an invitation to come to the Institute of Advanced Study (IAS) at Princeton. The only way real progress could be made was for Joe to move to Princeton and put his entire effort into working with several expert collaborators. It was a tempting offer and a great honor for Joe to be a scholar at the IAS. However, it never happened. Joe and Judy decided that such a change in lifestyle wasn't best for the family. A compromise was designed so that Joe would come down to Princeton once per week to spend the day and contribute what he could. Joe has great memories of that special year.

A working group that was a natural for Joe Arkin was the Fibonacci Society. Joe's first paper in the *Fibonacci Quarterly* was published in 1965. Joe met Vern Hoggatt through the Society and soon the two number theorists were producing results and publishing papers together. Joe attended the Fibonacci Society's international meetings. The most recent meetings he attended were in Winston-Salem and Scotland.

Joe Arkin desired to edit and publish a mathematics journal. Joe wrote a pamphlet in May 1991 entitled "Senior Lecturer's Problem Notebook." It's only issue contained an example problem, "Archimedes' Trisection of an Angle," explained by Joe; no elementary problems; and 4 advanced problems, 2 from Joe in number theory, and 2 from Paul Erdos involving properties of sequences. Joe was persistent and his publishing suggestions finally found an acceptable venue. He came up with an idea for a newsletter that would publish articles of interest to the mathematics departments of the service academies. Much of the newsletter would be for sharing teaching and curricular ideas, but there would be room for an occasional research article and description of research programs. *Mathematica Militaris* was born with Joseph Arkin as its Founding Editor. As evidenced by this issue, eight years and 25 issues later, it still going strong.

Joe finally had the opportunity to collaborate and co-author an article with Paul Erdos. Most of the work on this paper was done by mail. The manuscript was mailed back and forth a few times. On Aug 25, 1994, Erdos wrote back to Arkin: "I am a few days in the hills near Budapest in a hotel of the Academy; I was in Zurich 2 weeks ago. As far as I can tell, your paper with Colonel Arney is new and interesting; can you extend it for prime triples, p, p+m1, p+m2." Later Erdos approved the paper title and journal submission. After its publication in 1996, Joe Arkin held the honor of being an Erdos 1 (published directly with Erdos). He had already been a Straus 1 and an Einstein 2.

Joseph Arkin had been appointed Senior Lecturer at West Point and for eight years had collaborated with and mentored many faculty members in the Department of Mathematical Sciences. He retired from that position on 23 September 1994.

Joseph Arkin has written over 50 articles which have appeared in numerous publications such as the *Mathematics Magazine, Fibonacci Quarterly, SIAM Review, Duke Mathematical Journal, Journal of Recreational Mathematics, Notices of the American Mathematical Society, Canadian Journal of Mathematics, Pacific Journal of Mathematics, and Mathematics and Computer Education*. Among his co-authors and collaborators are many distinguished mathematicians and scientists including Paul Erdos, Ron Graham, E.G. Straus, Richard Pollack, Vern Hoggatt, Paul Smith, V.E. Smith, Gerald Bergum, and Stephan Burr. His co-authors at West Point include Bruce Porter, William Ebel, Charles Kennedy, Edith Luchins, Lee Dewald, Rick Kolb, Frank Giordano, and Chris Arney. His work is foundational in many areas and is, therefore, cited often in works of other researchers. Joseph Arkin has made over 50 presentations at professional meetings. He has attended and presented papers at the American Mathematical Society meetings, the meetings of the Metropolitan Section of the Mathematics Association of America, International Conferences on the Fibonacci Numbers, numerous number theory conferences, and Army Conferences on Applied Mathematics and Computing. Joseph Arkin has been a member of the New York Academy of Science, the Canadian Mathematics Society, and The Calcutta Mathematics Society. Some of Joseph Arkins, significant results are in the form of extensions and generalizations of earlier classical works of great mathematicians. Some of his papers of this type are: "An Extension of a Theorem of Ramanujan," "A Note on a Theorem of Jacobi," "Researches on Some Classical Problems," Exploded Myths," "New Observations on Fermat's Last Theorem," "On Euler's Solution to a Problem of Diophantus," and "An Extension of E. B. Straus' Perfect Latin 3-Cube of Order 7." Most of his work was performed in several related areas of number theory. In particular, Arkin made tremendous contributions in the following areas: Fibonacci and other recursive sequences, partitions, tilings, magic square and cubes, and Latin squares and cubes. His Army awards include the Certificate of Appreciation for Patriotic Civilian Service and the Commander's Award for Public Service.

Today, in this issue of *Mathematica Militaris*, we give special tribute to the many contributions made by Joe Arkin and his family. Joe lives at Ramapo Manor Nursing Home, Cragmere Road, Suffern, NY 10901. He's always happy to receive greetings from fellow mathematicians, so drop him a line and express thanks to a special person and a great mathematician.

New Tools for Computational Fluid Dynamics: An Overview

Dr. David J. Haroldsen, USMA

Over the past twenty years, the field of computational fluid dynamics (CFD) has dramatically grown and matured. The development of sophisticated mathematical algorithms and state of the art supercomputing capabilities has significantly improved the predictive capability of researchers studying computational fluid flow problems. The Army Research Laboratory is particularly interested in the study of projectile aerodynamics using CFD techniques. These problems involve sophisticated geometric designs and extreme and complex flow field conditions. An example of such a problem is a finned missile at angle of attack in supersonic flight.

For such complicated problems, there are several significant challenges: generation of computational grids around complex bodies, robust and accurate simulation of flow field variables, and effective processing and visualization of computational results. An important consideration which affects all three of these challenges is the need for computational tools are efficient in terms of both computational time as well as user time needed to operate the tools

My work with the Army Research Laboratory has involved investigating new approaches to the challenges of grid generation, flow solution, and flow visualization. As part of a joint project with other agencies, the current capabilities of various software tools were explored and compared. An aerodynamic problem of interest to the agencies was studied as a "model problem". My portion of the project involved working with a novel approach to grid generation, a comprehensive non-commercial flow solver, and new techniques for visualizing specific flow field features.

The grid generation package I used was GridPro, a commercial package developed by Program Development Corporation. GridPro uses a topology design approach to try to automate the grid generation process and enable the user to develop complicated multi-block grid structures in a relatively short time period. The design approach involves sketching out a rough outline of the desired grid topology and grid density. The software automatically places specific grid points while optimizing grid quality. The package requires some training to use, but offers the possibility of generating dense, high quality grids in a relatively short time period. In addition, because the process emphasizes topology rather than geometry, a single topology design can be used to generate grids for many different geometric shapes. Of interest for the study was the overall usability of the package as well as development of tools to export the grids to the flow solver and visualization packages of interest.

The flow solver for my study was WIND, developed by a consortium of government and private aerospace interests. The flow solver incorporates a broad spectrum of numerical techniques and physical models, including the most widely used turbulence models. WIND also incorporates a parallel processing capability. WIND is of interest largely because it has the capability to import grids generated by GridPro. Many multi-block flow solvers require some grid overlap at block boundaries. WIND uses coupling algorithms at block boundaries to avoid the necessity of block interface overlap. This is particularly important where complicated topological grid features require singularities in block interfaces. In this instance, creating a grid overlap is an ill-posed problem that requires significant additional resources to solve.

The visualization package used was pV3, developed by Robert Haimes at the Massachusetts Institute of Technology. The pV3 package incorporates a number of features of current interest, including the ability to detect flow features such as vortex core lines and shocks. An additional capability which was explored was the ability to couple the visualization package with the flow solver to allow real time visualization of flow field development.

The emphasis of this project was not to develop new products, but to use recent developments in new and novel ways to improve the predictive capability of researchers at the Army Research Laboratory. The GridPro software in particular has the potential to dramatically improve the ability to quickly generate large, complicated, high-quality grids for use with the laboratory's flow solvers. The WIND package proved to have many excellent capabilities, but requires further development to be of practical use. There were difficulties with robust calculation of solutions and with parallel efficiency. However, WIND is the subject of ongoing development, so it has the potential also to be an important tool in the future. The visualization package provided new insights into flow field characteristics and development. Future work envisioned includes using these and similar innovations to investigate even more complicated problems, including multiple body studies and transient flow problems.

Acknowledgements: The author expresses thanks to Dr. Walter B. Sturek of the Army Research Laboratory, Peter Eismann of Program Development Corporation, and Robert Haimes of MIT for valuable assistance, suggestions, and recommendations.

A Numerical Investigation of Supersonic Jet Interaction for Finned Bodies

Dr. Mary Jane Graham, USMA
Dr. Paul Weinacht, ARL and USMA

A detailed numerical investigation of the interaction between a lateral jet and the external flow has been performed for a variety of missile body geometries. The missile geometries include non-finned axisymmetric bodies and finned bodies with either strakes or aft-mounted tail fins. The computations were carried out at Mach numbers 2, 4.5 and 8. To obtain the numerical results, both Reynolds-averaged Navier-Stokes and Euler techniques have been applied. The computational results were compared with results from a previously published wind tunnel study that consisted primarily of global force and moment measurements. The results show significant interactions of the jet induced flowfield with the fin surfaces which produce additional effects compared with the body alone. In agreement with experiment in some cases the presence of lifting surfaces resulted in force and/or moment amplification of the jet interaction with the missile surfaces. The results indicate deamplification of the jet force at Mach 2 for all three bodies. Amplification of the jet force was also observed for high Mach numbers, particularly for the body with strakes. For the results examined here, there were only minor differences in the global force and moment predictions using viscous or inviscid techniques. The dependence of the interaction parameters on angle of attack and jet pressure were well predicted by both methods. The numerical techniques showed good agreement with the experiments at supersonic Mach numbers but only a fair agreement for the hypersonic, Mach = 8 case.

The flow field that results from the interaction of a side (lateral) jet injection into a supersonic external flow, called the jet interaction flow field, has been the subject of several experimental [2-5] and numerical [6-11] investigations. The typical jet interaction flow field is complicated due to the jet's interruption of the oncoming external flow. The qualitative features of the jet interaction flow field include regions of shock/boundary layer interaction and flow separation that have an effect on the overall flow around the body. In our previous work [11] a detailed numerical study was performed for non-finned axisymmetric bodies. In this paper, results are presented for missiles with several body geometries. The finned missile configurations are body strakes and aft-mounted fins. It was shown previously [11] that for a finless body, deamplification occurred partially because the jet bow shock wrapped around and interacted with the flow underneath the body. The presence of strakes has the effect of blocking the wrap-around phenomena and channeling the high pressure flow down the body therefore allowing for amplification. The purpose of the current research is to develop a reliable computational capability to assess the performance of control jets and to obtain a quantitative understanding of the flow phenomena produced by control jets in the presence of strakes and/or fins and to demonstrate that computational fluid dynamics (CFD) simulation is capable of predicting the important features of jet interaction phenomena.

This paper primarily addresses viscous techniques, but we have also examined the ability and feasibility of using inviscid techniques to predict the

same forces and moments. Numerical predictions of the supersonic viscous flow have been obtained using an existing Reynolds-averaged Navier-Stokes solver, and the inviscid flow simulations were performed with athree-dimensional multi-zone Euler technique. This work demonstrates that numerical techniques are sufficiently mature to be a useful predictive tool in design of jet-control systems and for flow diagnostics that cannot be made in the experiments. Comparisons between viscous and inviscid results can shed new light on the significance of viscous effects (i.e., separation of flow due to shock interaction) on overall vehicle forces and moments.

In addition to using several different geometries, the parameters that we vary in this study are Mach number, angle of attack, and jet stagnation pressure. The Mach numbers are 2.0, 4.5, and 8. The angles of attack range from -10 to 10 degrees. The jet stagnation pressure varies from 3.6 psi to 72 psi. It will be shown that the numerical results collaborate the experimental findings, in which the presence of strakes caused large control force amplification. This permits the use of smaller control jets and therefore results in propellant volume and weight savings.

In the present study, numerical approaches have been applied to investigate the jet interaction phenomena for flight bodies with lifting surfaces with a single lateral jet in supersonic flight and to demonstrate the advantages to force amplification factor in the presence of these surfaces. An overset grid approach has been applied to more easily resolve the geometry and flow physics associates with the jet interaction problem. All the numerical results have been validated using global force and moment data from a recently published experimental investigation [5].

Computational Technique

Full details of the numerical technique used to solve the time-dependent Navier-Stokes equations are found in "Effect of Fin Span on lateral Control Jet Effectiveness," Brengle, John (CDT) this Publication.

Navier-Stokes Numerical Technique

The time-dependent Navier-Stokes equations are solved using a time-iterative solution technique to obtain the final steady-state converged solution. The particular time-marching technique applied here is the implicit, partially flux-split, upwind numerical scheme developed by Steger, et al [15,16], and is based on the flux-splitting approach of Steger and Warming [18]. This scheme utilizes central differencing in the normal, η, and circumferential directions, η and ς, respectively; and flux splitting in the streamwise direction, ξ.

Chimera Composite Overset Structured Grids

The full computational grid can be seen in Fig. 1. It is comprised of a main grid for the flight body and an overset grid. The overset grid is decomposed into two pieces: one for the nozzle and one for the jet. A close-up of this grid is in Fig. 3 of CDT John Brengle's paper in this publication. Communication between the grids for the jet and the body is performed by the Chimera scheme (details can be found in Brengle's paper.)

Inviscid Technique

The flow field solution is obtained here by running the CFD code named INCA [21]. The code is presently run in the Euler (inviscid) mode, but INCA is a multi-block, Navier-Stokes solver with broad capabilities. The field equations are solved using an implicit finite-volume method. The evaluation of the inviscid terms is based on flux splitting in combination with upwind biasing.

The boundary condition specified on all in-flow boundaries was the supersonic in-flow condition that specifies the velocity components, pressure, and temperature of the flow. The outflow is expected to be supersonic as well, and the supersonic outflow condition is used. Here, flow conditions cannot be specified *a priori,* but they are extrapolated from the upstream cells adjacent to the boundary cells. Along the body surface tangential full-slip flow is specified, since the flow is inviscid. At the vertical (pitch) symmetry plane, the symmetry conditions (mirror image) is specified, as only half of the flow field is computed. The jet flow is specified as supersonic in-flow over the patch of cells defining the nozzle. The sonic in-flow in the direction normal to the freestream direction, and jet exit pressure and temperature are specified here. The initial conditions used to start the CFD are specified within the whole computational domain using the ambient flow parameters.

Results

Validation of the computational approach for the jet interaction problem was accomplished by comparing the predictions with data from a previously published wind tunnel investigation [5]. Supplemental experimental results for the validation were provided courtesy of Rafael/Ministry of Defense, Directorate of Defense Research & Development. The experiments were conducted at the Israel Aircraft Industries (IAI) trisonic wind tunnel facility at Mach numbers of 2 and 4.5 and in the IAI hypersonic wind tunnel facility at Mach 8. While the experimental investigation was quite extensive in scope, the current computational study focused on normal jet injection from a single nozzle geometry at Mach 2, 4.5, and 8. Five different configurations were examined in the experimental study; we examine three. Global force and moment comparisons were made to validate the computational approach.

Fig. 1 in Brengle's paper shows three body geometries addressed in this study. Each of the models used has a sharp, ogive-shaped nose section of 2 calibers and a cylindrical afterbody of 3.3 calibers mounted to the midsection for the total length of 5.8 calibers. The reference diameter of the models was 50 mm. For all three geometries, the jet nozzle was located 2.5 calibers downstream from the nose tip. A single 5-mm circular nozzle that was designed to achieve sonic flow at the exit was examined here, although additional geometries were considered in the experiment. The strake and aft-mounted fins have the same exposed semi-span of 25~mm and the leading edge sweep angle of 45 degrees. Configuration 1 is an axisymmetric body-alone configuration used as a reference configuration for comparison. Configuration 2 has an aft-mounted tail fin. The root leading edge of each tail surface is located 220~mm from the nose tip. Configuration 3 contains strakes spanning 65 percent of the body's length. The root leading edge is 100~mm from the nose tip. Global force and moment wind tunnel tests were performed on these bodies.

For the jet interaction problem, the total force acting on the body can be decomposed into three components: the aerodynamic force on the external body in the absence of the jet, the force produced at the jet exit, and the aerodynamic interaction force produced by the jet with the external flow field. In this work, the aerodynamic force on the external body is typically produced when the body is at an angle of attack with the freestream flow. The force produced at the jet exit results from a combination of the momentum flux through the jet nozzle and the integrated pressure at the jet exit. Given that the exit conditions for the jet are fixed as a boundary condition for the computations, this force component can be explicitly calculated prior to the flow field computation. The third force component accounts for the force produced by the interaction of the jet with external flow field.

The relationship of these three force components to the total force, F, can be described by the following equation, where F_{no-jet} is the force in the absence of the jet, F_j is the force produced at the jet exit, and F_{ji} is the jet interaction force.

$$F = F_{no-jet} - (F_j + F_{ji})$$

The negative sign associated with the two jet forces results because the jet exit hole is located on the upper surface of the body in the current study and produces a downward force when activated. The jet-off force component typically produces an upward force for positive angles of attack. Note that a positive value of F_{ji} indicates that the interaction force produces an effect that augments the jet force F_{ji}, while a negative value of F_{ji} indicates a reduction in the total force produced by the jet. The jet interaction force accounts for the complete interaction produced by the jet with the external flow field and may vary with angle of attack and jet mass flow rate.

The relative magnitudes of the jet force and the jet interaction force can be compared through a jet interaction amplification factor, K, as shown in the following equation:

$$K = \frac{F_j + F_{ji}}{F_j}$$

An amplification factor greater than 1 indicates that the jet interaction force amplifies or increases the total force produced by the jet, while an amplification factor less than 1 indicates that the jet interaction force reduces the total force produced by the jet.

Figs. 2-4 display the variation of the force amplification factor with angle of attack at Mach 2, 4.5 and 8 for the body alone and body with tail fins. This comparison is meaningful because ahead of the tail fins (for these supersonic flow cases), the force amplification is essentially identical for both bodies. The differences in the force amplification factor occur only over the aft 1.4 calibers of the body which contains the fins. The predicted results at Mach 2 and 4.5 show excellent agreement with experiment between -10 and 10 degrees angle of attack. At Mach 8, the results show an underprediction of the jet amplification factor, though the trend with attack of attack is consistent with the experimental data.

The results for the body with tail fins shows an increasing trend with angle of attack. For positive angles of attack, the force amplification factor for the tail fins is similar in magnitude of the body alone results. However, at negative angles of attack, the body with tail fins shows a much stronger deamplification than for the body alone. The differences in the behavior at positive and negative angles of attack are due to interaction of the jet wake on the tail fins. At positive angles of attack, the jet wake is directed away from the tail fins, while at negative angles of attack, the jet wake is convected downwards onto the tail fins producing stronger interaction and deamplification.

Ahead of the tail fins, the distribution of the jet interaction force are identical to the body alone results. Very little additional interaction over the tail fins is seen at $\alpha = 10$. However, at $\alpha = -10$, there is a significant interaction over the tail fins which results in deamplification.

An increasing trend in force amplification factor is noted with increasing Mach number in Figures 2-4. However, the force amplification factor also varies with jet pressure. No attempt to scale the jet pressure has been made for the results presented in Figures 2-4.

Figs. 5-7 display the variation of the force amplification factor with angle of attack at Mach 2, 4.5 and 8 for the straked body. At Mach 2 and 4.5, the results are in good agreement with the experimental data across the range of angles of attack. At Mach 8, the computed results do not compare as well with the experimental data. At Mach 4.5 and Mach 8, results were obtained with an inviscid code, as well as with the Navier-Stokes code. At Mach 4.5, there is very little difference in the predicted results with either code. Similar results are found at Mach 8, where both codes show a trend which is different than the experimental data. The similarity between the predicted results is significant considering the results were obtained with different codes and computational grids.

A comparison of the jet interaction force distribution for the body alone and body with strakes at zero degrees angle of attack at Mach 2, Mach 4.5 and Mach 8 was performed. For all three
Mach numbers, the presence of the strakes amplifies the jet interaction effect. Near the jet exit, the high pressure behind the jet bow shock acts not only on the body, but on the adjacent fins as well. This produces an additional force augmentation relative to the body alone. At Mach 2, the effect of the low pressure region behind the jet is also increased for the straked body producing a force component that results in a deamplification of the jet force.

The force amplification factor for the straked body also shows an apparent increasing trend with Mach number as did the results for the body alone and body with tail fins shown previously. Again, it may be difficult to draw a general conclusion from these results alone because of the dependence of the amplification factor on jet pressure.

Summary and Conclusions

A computational approach has been validated with experimental data for bodies with lateral control jets
in supersonic flight at varying Mach numbers and angles of attack and for different jet stagnation pressures. The bodies include a body alone configurations, a body with aft-mounted tail fins and a body with strakes. The results show significant interactions of the jet-induced flowfield with the fin surfaces which produce additional effects compared with the body alone. In agreement with experiment in some cases the presence of lifting surfaces resulted in force and/or moment amplification of the jet interaction with the missile surfaces. The results indicate deamplification of the jet force at Mach 2 for all three bodies. Amplification of the jet force was also observed for high Mach numbers, particularly for the body with strakes. For the results examined here, there were only minor differences in the global force and moment predictions using viscous or inviscid techniques. This similarity indicates that the viscous effects are small for these configurations, especially in view of the large lifting surfaces that receive the pressure forces. Both techniques correctly predicted the dependence of the interaction parameters on angle of attack and jet pressure. The results indicate that for the purpose of overall design of configurations with jet force control, the inviscid methods may be both sufficient and expedient. Viscous computations are, however, imperative when the near field close to the jet is considered.

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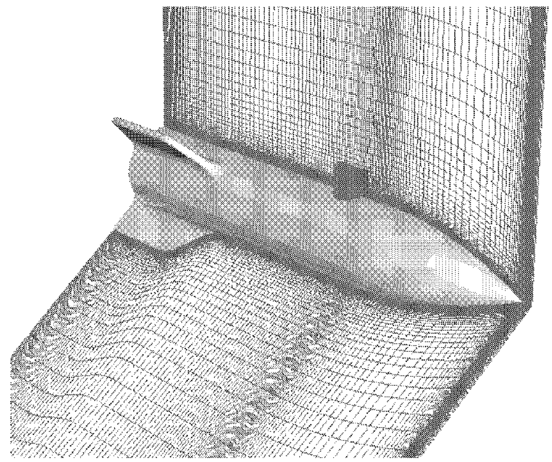

Fig. 1 Computational Mesh.

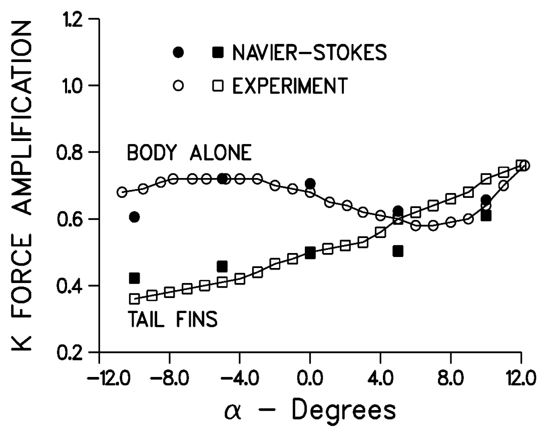

Fig. 2 K vs. alpha, Mach 2.0.

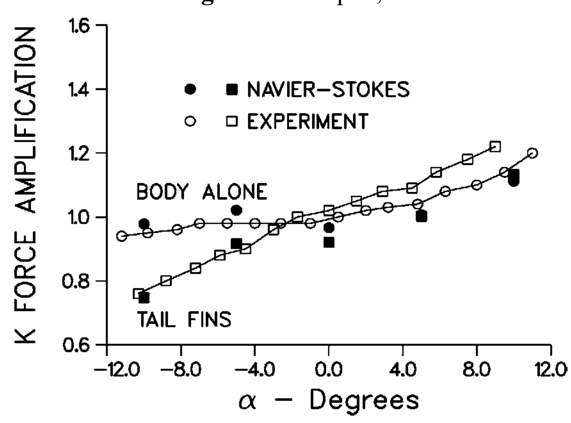

Fig. 3 K vs. alpha, Mach=4.5

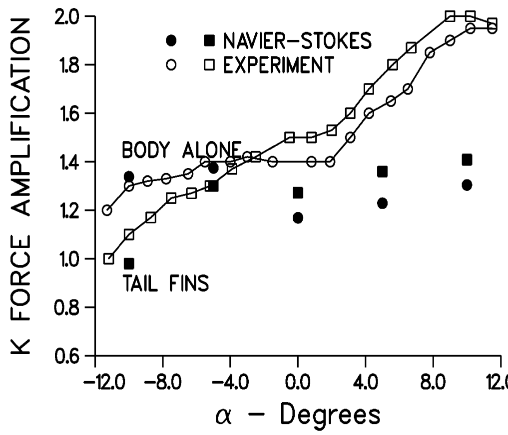

Fig. 4 K vs. alpha, Mach=8

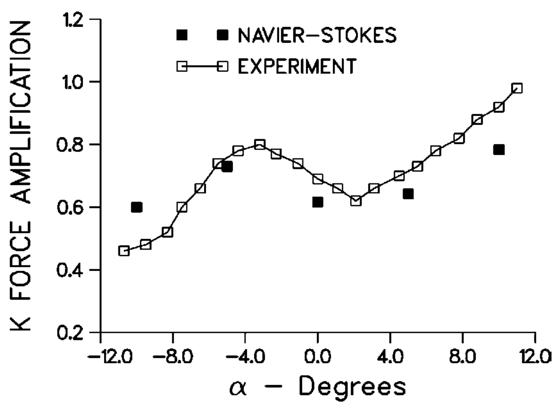

Fig. 5 K vs. alpha, Mach=2.0

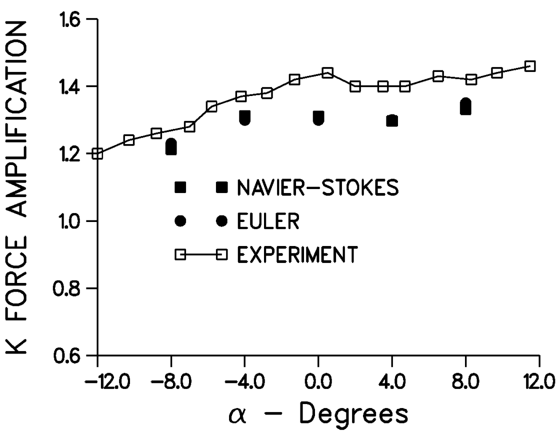

Fig. 6 K vs. alpha, Mach=4.5

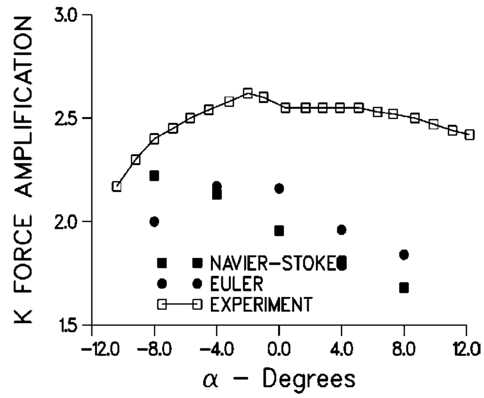

Fig. 7 K vs. alpha, Mach=8

Effect of Fin Span on Lateral Control Jet Effectiveness

CDT. J. W. Brengle, USCC, USMA

Sponsors of this work are Drs. M. J. Graham and P. Weinacht, USMA and ARL, respectively.

Abstract

Previous research has shown that for axisymmetric flight bodies, the presence of lifting surfaces (fins or strakes) increases the effectiveness of lateral control jets. Recent wind tunnel testing and numerical simulations have shown that for non-finned axisymmetric bodies with lateral control jets in use, a deamplification of the jet interaction force occurs. Further studies have shown that the presence of fins or strakes along a flight body with lateral control jets yields an amplification of the jet interaction force. These studies investigated the jet interaction force for different fin geometries with a constant fin span. To further advance the research in this area, we present a study of the affect on the jet interaction force by changing the fin span of the strakes along the flight body. Our results were obtained by applying numerical simulations based on Reynolds-averaged Navier-Stokes techniques. This investigation was conducted for a constant flight velocity of Mach 4.5 while varying the fin strake height (0.0, 0.125, 0.25, 0.375, 0.5, and 0.75 body diameters from fin root to fin tip). This would equate to Configuration 3 found in Figure 1. The computational results showed that the force amplification factor increases as the fin span increases. For the particular jet exit conditions examined here, the force amplification factor shows continues to increase with increasing fin span even for very large fin spans.

Introduction

The flow field that results form the interaction of a side (lateral) jet injected into a supersonic external flow, called the jet interaction flow field, has been the subject of several experimental[2-5] and numerical[6-11] investigations. The typical jet interaction flow field is complicated due to the jet's interruption of the oncoming external flow. The qualitative features of the jet interaction flow field include regions of shock/boundary layer interaction and flow separation that have an effect on the large regions of the flow field around the body. In previous work[11], a detailed numerical study was performed for non-finned axisymmetric bodies. The computational fluid dynamic (CFD) model based on Reynolds-averaged Navier-Stokes numerical techniques has been successful in providing results that adequately predict the experimental results.[12] In this paper, results are presented for straked bodies with varying fin strake height. It was shown previously[11] that for a finless body, deamplification occurred partially because the jet bow shock wrapped around and interacted under the flight body. The presence of strakes has the effect of blocking the wrap-around phenomena and channeling the flow down the body therefore allowing for amplification. The purpose of the current research is to investigate the relationship between the fin strake height and the force amplification factor, K. The force amplification factor is defined as

$$K = \frac{F_{jet} + F_{ij}}{F_{jet}} \qquad (1)$$

where an amplification factor greater than 1 indicates that the jet interaction force amplifies or increases the total force produced by the jet, while an amplification factor less than 1 indicates that the jet interaction force reduces the total force produced by the jet.

This study investigated how K varied at Mach 4.5. The parameter that varied for this study was the fin strake height. The height ranged from 0.0D to 0.75D where D is the projectile diameter. Figure 2 shows three of the six straked body geometries used in the investigation.

Computational Technique

Governing Equations

The nonreacting compressible Newtonian viscous flow about a fluid vehicle is governed by the equations of mass, momentum, and energy conservation: The Navier-Stokes equations. For these computations, the complete set of three-dimensional, time-dependent, generalized-geometry, Reynolds-averaged, thin-layer, Navier-Stokes equations for generalized coordinates ξ, η, and ζ are used and can be written as follows:[13]

$$\frac{\partial \hat{q}}{\partial t} + \frac{\partial \hat{E}}{\partial \xi} + \frac{\partial \hat{F}}{\partial \eta} + \frac{\partial \hat{G}}{\partial \zeta} = \frac{1}{\mathrm{Re}} \cdot \frac{\partial \hat{S}}{\partial \zeta} \qquad (2)$$

$\xi = \xi(x, y, z, t)$, $\eta = \eta(x, y, z, t)$, and $\zeta = \zeta(x, y, z, t)$ are the longitudinal coordinate (direction along the body, the circumferential coordinate (direction around the body), and the nearly normal coordinate (outward direction from the body surface) respectively.

The inviscid flux vectors, $\hat{E}$, $\hat{F}$, and $\hat{G}$, and the viscous term $\hat{S}$ are functions of the dependent variable $q^T = (\rho, \rho u, \rho v, \rho w, e)$. The inviscid flux vectors and the source term are shown as follows. Details of the thin-layer viscous term are available in the literature.

$$\hat{E} = \frac{1}{J}\begin{bmatrix} \rho U \\ \rho u U + \xi_x p \\ \rho v U + \xi_y p \\ \rho w U + \xi_z p \\ (e+p)U \end{bmatrix}$$

$$\hat{F} = \frac{1}{J}\begin{bmatrix} \rho V \\ \rho u V + \eta_x p \\ \rho v V + \eta_y p \\ \rho w V + \eta_z p \\ (e+p)V \end{bmatrix}$$

$$\hat{G} = \frac{1}{J}\begin{bmatrix} \rho W \\ \rho u W + \zeta_x p \\ \rho v W + \zeta_y p \\ \rho w W + \zeta_z p \\ (e+p)W \end{bmatrix} \qquad (3)$$

The contravariant velocity components (in the ξ, η, and ζ directions) that appear in the inviscid flux terms have the following form:

$$U = u\xi_x + v\xi_y + w\xi_z \qquad (4)$$

$$V = u\eta_x + v\eta_y + w\eta_z \qquad (5)$$

$$W = u\zeta_x + v\zeta_y + w\zeta_z \qquad (6)$$

The Cartesian velocity components (*u, v, w*) are retained as the dependent variables and are nondimensionalized with the respect to a_∞ (the freestream speed of sound). The local pressure is determined using an appropriate equation of state (i.e., the pressure is related to the dependent variables by applying the ideal gas law):

$$p = (\gamma - 1)\left[e - 0.5\rho(u^2 + v^2 + w^2)\right], \qquad (7)$$

where γ is the ratio of specific heats. Density, ρ, is referenced to ρ_∞ and total energy, e, to $\rho_\infty a_\infty^2$.

The form of the mass-averaged Navier-Stokes equations requires a model for the turbulent eddy viscosity. There are numerous approaches for determining the turbulent viscosity. The turbulent contributions are supplied through the algebraic two-layer eddy viscosity model developed by Baldwin and Lomax,[14] which is patterned after the model of Cebeci.[15]

Navier-Stokes Numerical Technique

The time-dependent Navier-Stokes equations are solved using a time-iterative solution technique to obtain the final steady-state converged solution. The particular time-marching technique applied here is the implicit, partially flux-split, upwind numerical scheme developed by Steger et al.[16,17] and is based on the flux-splitting approach of Steger and Warming.[18] In its original form, the technique was referred to as the F3D technique. This scheme utilized central differencing in the normal and circumferential directions, η and ζ, respectively, and flux splitting in the streamwise direction, ξ. Rather than directly invert the implicit equation, a two-factor implicit technique similar to that of Steger and Buning[19] is utilized. The two-factor implicit algorithm involves two sweeps through the grid at each time step. The first sweep involves inverting a block tridiagonal system of equations η grid lines to determine the intermediate solution variable. During the second sweep, a second block tridiagonal system of equations is inverted along grid lines of constant ζ to determine the dependent variable. The two-factor implicit algorithm reduces the computational requirements of the approach compared with the three-factor, central difference, implicit algorithm of Beam and Warming.[17]

The algorithm contains additional numerical smoothing terms to suppress numerical oscillations associated with the odd-even decoupling produced by the central differencing in the η and ζ directions.[16]

Chimera Composite Overset Structured Grids

To more easily model the geometry and resolve the flow physics associated with the lateral jet problem, the Chimera composite overset grid technique has been applied. The Chimera technique is a domain decomposition approach that allows the entire flow field to be meshed into a collection of independent grids, where each piece is gridded separately and overset into a main grid. In current computation, the flight body with lateral jet was subdivided into three distinct grids: one for he body, one adjacent to the jet, and one for the jet nozzle. Overset grids are not required to join in any special way. Usually there is a major grid that covers the main domain (the external flow field about the projectile), and minor grids are generated to resolve the rest of the body or sections of the body (the jet and the nozzle regions).

Figure 3 displays the computational mesh, showing the main grid for the projectile body along with an overset grid to better capture the physics of the jet interaction with the external flow. The overset jet grid is seen here residing on top of the jet exit as a cylinder with a radius larger than the jet hole itself. A third grid, used to model the jet nozzle, resides underneath the jet grid. The communication between the nozzle grid and the jet grid, however, is point-to-point zonal. Figure 4 is a close up of the grid near the jet hole, which is covered by the nozzle grid and the jet grid. It also shows the Chimera grid for the jet and the projectile body. A hole has been cut into the projectile grid by the scheme. Because each component grid is generated independently, portions of one grid lie within the solid boundary contained in another grid. Such points lie outside the computational domain and are excluded from the solution process. Any viable structured grid flow solver can be adapted to work within the framework of the Chimera scheme.

Results and Discussion

With each incremental increase in the fin strake height, the force amplification factor increased as well. Figure 5 illustrates the almost linear relationship between the two quantities. Figure 6 illustrates how the jet interaction force coefficient increases as well with each incremental change in fin strake height. The results indicate that as the strake height increases, more of the high-pressure energy is captured between the strakes thus increasing the normal force felt by the projectile. So long as the strakes increase in height, their surface area will increase and provide a larger area for the pressure force to act upon. It is suspected that this phenomenon will subside when the flow field pushes the effects of the jet behind the strakes before they can act upon the projectile. This would either happen as very great speeds or when the strake height is very great. Both conditions, we speculate, are not practical to construct for purposes other than research because drag forces will limit the height of the strake based on an optimal configuration. The force amplification factor does not appear to have an optimal value based on strake height alone.

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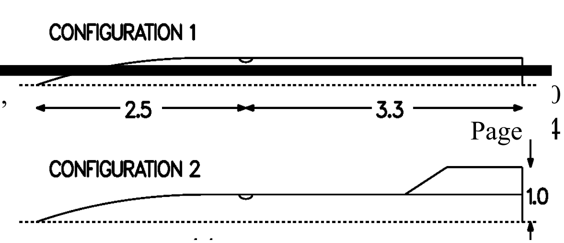


Fig. 1 Schematic of body geometries.

Fig. 2 Straked Body Geometries.

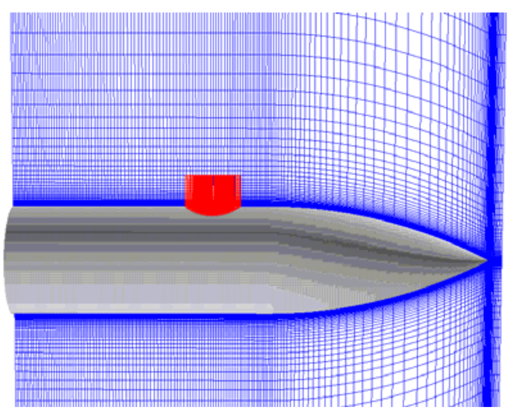


Fig.3 Computational Mesh

Fig. 4 Chimera gridding near jet nozzle.

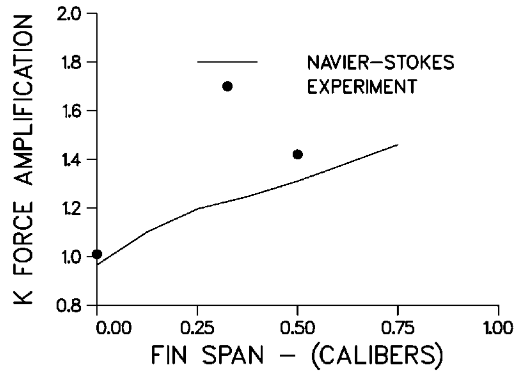


Fig. 5 Force amplification factor as a function of fin strake height, Mach 4.5.

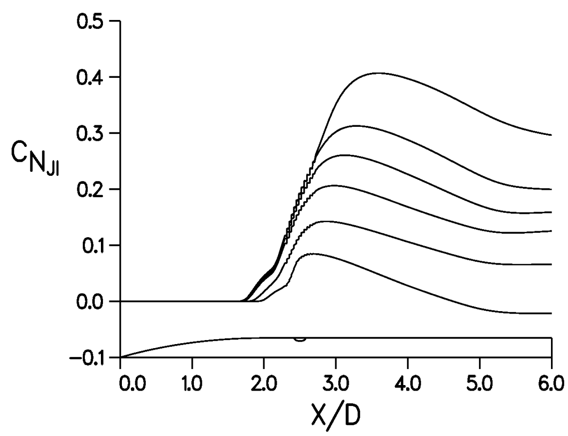
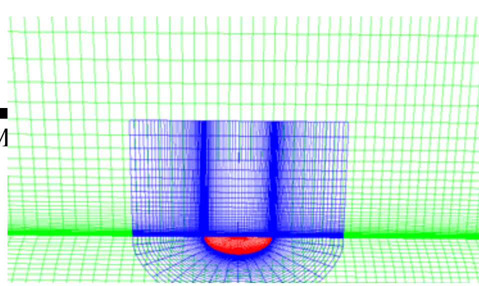


Fig. 6 Distribution of JI force over body for various fin strake heights.



The Operations Analysis Course at the

U.S. Coast Guard Academy

LT. Mary Jo Meilstrup, USCGA

The Operations Analysis course serves as the capstone course for the Operations Research major and is presented as a project-oriented course during the spring semester of the cadets' senior year. It is an opportunity for cadets to put into practice what they have learned in the classroom. Consulting teams of cadets are assigned to projects submitted by various Coast Guard units and approved by the Head of the Department of Mathematics. Cadet teams are required to work with project sponsors to define the problem to be investigated and to use appropriate statistical and operations research techniques to solve the problem. They must submit a written report to both the project sponsors and the course coordinator, and make an oral presentation to the sponsors, faculty in the Department of Mathematics, underclass cadets in the Operations Research major, and invited guests from the Institute for Operations Research and the Management Sciences.

Projects generally require cadets to access Coast Guard databases and to interface with personnel at various Coast Guard commands. Cadets utilize a number of software packages including *Microsoft Access*, *Microsoft Excel*, and *Data Desk*, as well as other statistic, forecasting, and simulation packages. Naturally, many of the projects assigned to cadet consulting teams have social, political, and economic implications that must be addressed as part of their investigation. Because of the complex nature of the consulting projects assigned, teamwork is required, and cadets must effectively utilize sound organizational skills to produce a polished finished product.

During the spring of 1999, cadets were engaged in consulting efforts to analyze search and rescue data and identity factors that contributed to successful Coast Guard missions. Another team of cadets worked with drug law enforcement data to determine which factors have the most significant impact on the total number of drug interdictions in the Coast Guard's Seventh District. A third group of cadets designed a data collection instrument for the Coast Guard's Research and Development Center. This instrument was an internet survey designed to collect data from mariners regarding methods of navigation and aid usage. A fourth team developed an optimal storage configuration of External Bulk Material units capable of withstanding forces created by extreme winds for the Coast Guard's Aircraft Repair and Supply Center (ARSC). The results of this last project were tested frequently during the fall hurricane season.

During the spring of 2000, one team developed a model that determined the best indicators to forecast when losses in the Coast Guard's enlisted workforce will occur throughout the year. Another project entailed the development of a model that determined the upper and lower bounds of ideal officer year groups considering voluntary loss rates, non-voluntary loss-rates, billets, promotion points, and opportunity of selection and accession mix. A third project involved the development of a Recruit Training schedule for Coast Guard Training Center Cape May in support of the revised Apprentice Curriculum and 9.5-week recruit training program. A fourth team of cadets constructed an analysis of the Ready-For-Issue material at ARSC. The cadets analyzed quantity on hand, demand history, available issue/receipt information, and any associated trends. Finally, a fifth team of cadets developed statistical models allowing the Coast Guard's Office of Shore Activities to predict changes in multi-mission boat station response based on budget and/or resource-driven changes.

Feedback provided by the sponsoring activities, the course coordinator, faculty mentors, and invited experts is used to evaluate the projects with regard to technical merit and usefulness to the sponsoring activity. The feedback provided by the various evaluators is also utilized to evaluate the competencies of the Operations Research majors. As such, this feedback enables the faculty of the Department of Mathematics to identify the strengths and shortcomings in the Operations Research curriculum. As importantly, by linking the Operations Research curriculum to ongoing projects in the Coast Guard, faculty in the Department of Mathematics are positioned to make changes to the curriculum in response to the needs of the service. The connectivity to the operational Coast Guard provided by the Capstone Course allows the Operations Research curriculum to remain current and vital.

Mathematical Research Programs at West Point in Support of the Army

LTC Philip Beaver, USMA
CPT Douglas Matty, USMA
LTC Michael D. Phillips, USMA

INTRODUCTION

Mathematical research in the Department of Mathematical Sciences at USMA can be described as varied and diverse. We have many research opportunities in the areas of teaching and learning, mathematical history, and applied mathematics, and all of our faculty members are encouraged to take advantage of these opportunities for personal and professional development and to enhance our classroom teaching with personal research experiments. Additionally, particularly for our rotating military faculty, these opportunities should allow the faculty members to remain current with Army issues and Army problems, and at the same time allow Army agencies to take advantage of a valuable research resource that we can provide.

The research opportunities we provide for solving Army problems come in three flavors: Research in support of the U.S. Army Research Laboratory (ARL), The Operations Research Center (ORCEN), and "others." Here we give a brief synopsis of each of these programs, along with examples of the type of work done under each one.

ARMY RESEARCH LABORATORY OPPORTUNITIES

In 1993, the Director of the U.S. Army Research Laboratory (ARL) and the Dean of the Academic Board at USMA signed a Memorandum of Agreement establishing the Mathematical Sciences Center of Excellence (MSCE) in the Department of Mathematical Sciences. The MSCE serves as ARL's focal point for identifying, evaluating, assigning, and supervising collaborative research areas with USMA's 9 Science and Engineering Departments and 14 Research Centers. Together, ARL and USMA cosponsor the following research programs**.**

- *ARL Visiting Scientist Chair and Researcher Chair.* These positions within the MSCE offer unique opportunities for professional development, as well as opportunities to establish strong research ties between USMA and ARL.
- *Davies Postdoctoral Fellowship*. The National Research Council (NRC) in concert with ARL and USMA established the Davies Postdoctoral Fellowship in 1995. This program provides an opportunity for recently graduated doctoral students to serve a three-year term concurrently in an academic department at USMA and a research laboratory at ARL. Currently, there are five Davies Fellows in the program involving three academic departments: Mathematical Sciences, Civil & Mechanical Engineering, and Physics.
- *ARL/USMA Cadet & Faculty Research Program and the Annual ARL/USMA Technical Symposium*. The MSCE provides a focal point for research activities in the mathematical sciences that extends beyond the Department of Mathematical Sciences and USMA, with the hope of developing lasting outreach affiliations. These outreach activities involve all USMA departments. In 1999, USMA cadet and faculty researchers from the departments of Mathematics, Physics, Electrical Engineering and Computer Science, Civil and Mechanical Engineering, Chemistry and Life Sciences, Foreign Languages, and Behavioral Science and Leadership conducted collaborative research with ARL through the MSCE. Annually, USMA faculty (military and civilian) and cadets to present and discuss various aspects of their collaborative research at the ARL/USMA Technical Symposium. Since the program began in 1993, over 90 faculty members and numerous cadets have conducted research with ARL. Below is a sample of the research conducted in AY 2000.

"Effect of Fin Span on Lateral Control Jet Effectiveness"
Cadet John Brengle, United States Corps of Cadets;
Dr. Mary Jane Graham, (Mathematical Sciences Davies Fellow), Department of Mathematical Sciences, USMA; Dr. Paul Weinacht, ARL Visiting Scientist, Weapons & Materials Research Directorate, ARL, Mathematical Sciences Center of Excellence.

"Using the Effective Index Method to Model Vertical-cavity Surface-emitting Lasers"

Dr. Keith Aliberti (Physics Davies Fellow), Department of Physics, USMA; Dr. Paul Shen, Sensors & Electron Devices Directorate, ARL.

"Effect of RGB Stereo Sensor Fusion on Target and Range Identification"

Cadet Anthony Marinos, United States Corps of Cadets; Dr. Wendel Atkins, Survivability/Lethality Analysis Directorate, ARL; LTC Michael D. Phillips, Mathematical Sciences Center of Excellence, Department of Mathematical Sciences, USMA.

"Vulnerability of Structural Panels to Blast Effects"

Dr. Chris Conley, Dept. of Civil & Mechanical Engineering, USMA; Mr. Fred Gregory, Weapons & Material Research Directorate, ARL.

"Automated Voice Recognition Systems For an NBC Environment"

LTC Jose A. Picart, Dept. of Behavioral Sciences & Leadership, USMA; Dr. Sehchang Hah, Dept. of Behavioral Sciences & Leadership, USMA; Dr. Ronald A. Weiss, Survivability/Lethality Analysis Directorate, ARL.

OPERATIONS RESEARCH CENTER

The United States Military Academy's Operations Research Center (ORCEN) provides a small, full-time analytical capability in support of the Academy's purpose and mission, the goals of the academic program and the disciplines of the systems engineering, operations research and engineering management. The ORCEN is organized under the Office of the Dean as an Academy Center of Excellence. It typically employs five full-time Army Analysts; at any point in time, about half a dozen Department of Systems Engineering and Mathematical Sciences military and civilian faculty, together with students of the military Academy, are working on a part-time basis on ORCEN projects. The ORCEN is collocated with the Department of Systems Engineering in Mahan Hall, West Point, NY and is sponsored by the Assistant Secretary of the Army (Financial Management).

The goals of the Operations Research Center include: enrich cadet education; enhance the professional development opportunities of the Academy faculty by providing opportunities to engage in current issues and areas of importance to the Army; establish and maintain strong ties between the Academy and the Army; and remain abreast of and integrate new technologies into academic programs. Fully staffed and funded since Academic Year 1991, the ORCEN has made significant contributions to the Army's analytical efforts.

Several areas of work contributed by ORCEN analysts included conducting the needs analysis and performance measures for unmanned aerial vehicles to support the Army's Force XXI initiative; develop and validating mission success templates for use during the Comanche Test and Evaluation to translate system performance into force success; determination of the optimal mix of aviation assets for the force structure of an aviation task force. Current work being conducted in the ORCEN includes supporting agencies in Headquarters, Department of the Army on such issues as the Manning Task-force to improve current human resource allocation to improve unit readiness and provide project management and analytical support to the Army Development XXI Task-force to reengineer the Military Human Resource Management System to improve how the Army manages its 480,000 soldiers.

OTHER RESEARCH OPPORTUNITIES

For rotating military faculty, most of the research opportunities in support of Army agencies come during the Summer, although many are begun or extended into the academic year. (The other Summer of the West Point teaching assignment is generally spent supporting cadet field training.) The Department of Mathematical Sciences has habitual relationships with some Army agencies (in addition to those mentioned above) but has sent officers to conduct research with a variety of agencies.

Among the places Department faculty have conducted research for the Army are DCSOPS, PERSCOM, Benet Laboratory, United States Special Operations Command, NASA, TRADOC Analysis Command, White Sands Missile Range, and several others. The Department has also had a habitual research relationship with the Army Digitization Office (ADO) since 1997.

Current ADO projects include an investigation of the utility of information on the battlefield, where

COL David C. Arney heads a team of researchers; an investigation of the optimal configuration of a communications net, where COL Gary Krahn and LTC Steven Horton head a team; and an analysis of the Army Flow Model where MAJ Gerald Kobylski heads a group of six investigators.

These opportunities, and others like them, help keep our faculty active with current Army issues, enhance the relevance of what we teach through integrating our research into our classroom instruction, and help the Army with solutions to some of its harder problems.